MATHEMATICS 7

Fractions and Decimal Numbers

Distance Learning

Alberta EDUCATION

Module 2

Mathematics 7

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FRACTIONS AND DECIMAL NUMBERS

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Mathematics 7
Student Module Booklet
Module 2
Fractions and Decimal Numbers
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This document is intended for	
Students	✓
Teachers (Mathematics 7)	✓
Administrators	
Parents	
General Public	
Other	

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Welcome


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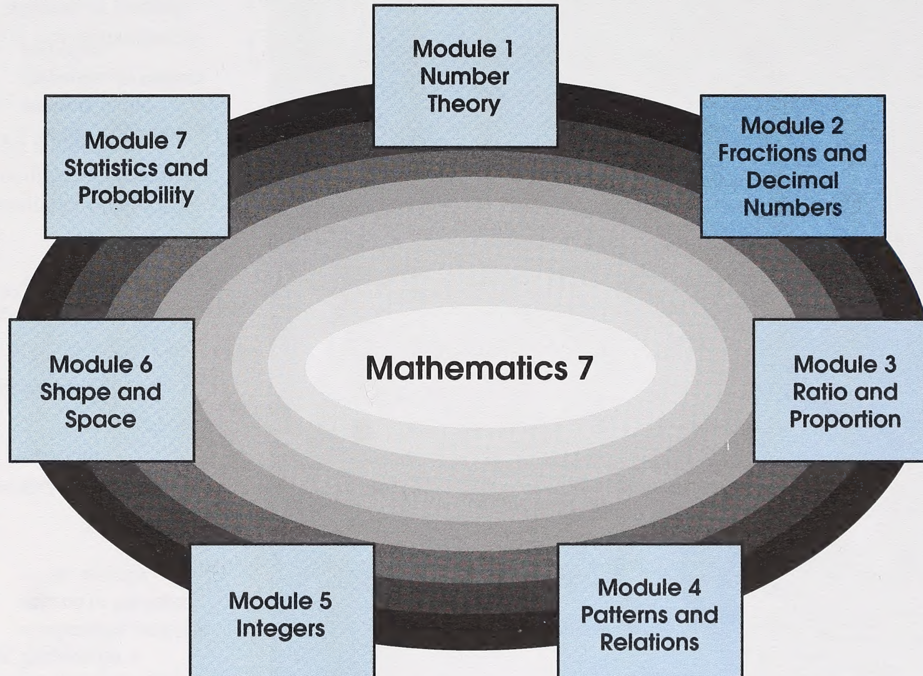
Welcome to Module 2. We hope you'll enjoy your study of Fractions and Decimal Numbers.

Mathematics 7 contains seven modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

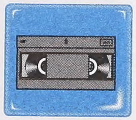
- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

- Use computer software.

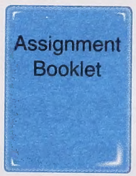
- Use a scientific calculator.



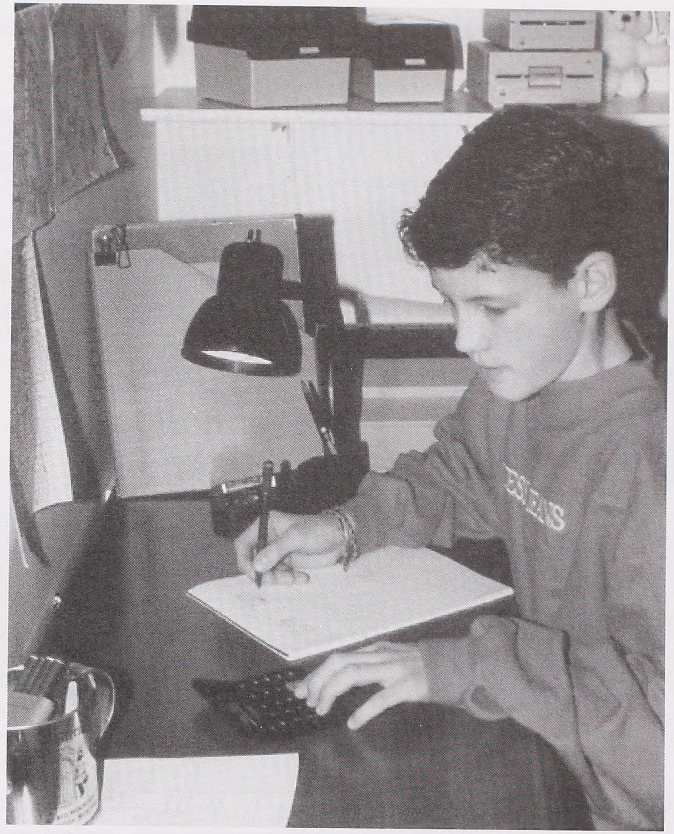
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge.

The icon is a cue that the problem will be related to the topic of the activity.

The icon is a cue that the problem will provide a change of topic.

The Four-stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem.

You may consider the following strategies:

- changing your point of view
- using objects
- using diagrams
- making an organized list
- using Venn diagrams
- making a table
- guessing, checking, and revising
- acting out a problem
- working backwards
- simplifying a problem
- finding and applying a pattern
- using elimination
- using truth tables
- using an equation

Note: The Appendix in Module 1 explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to the Appendix in Module 1 and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

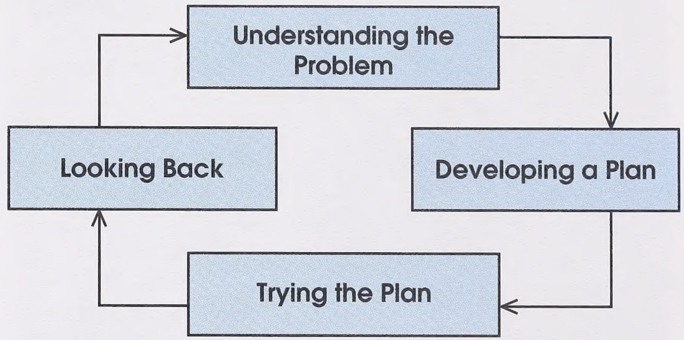
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

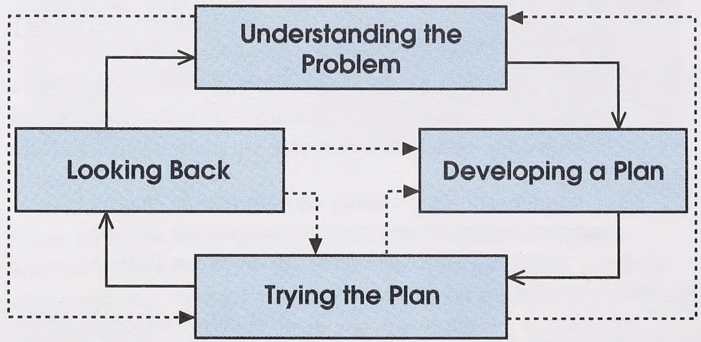
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



CONTENTS

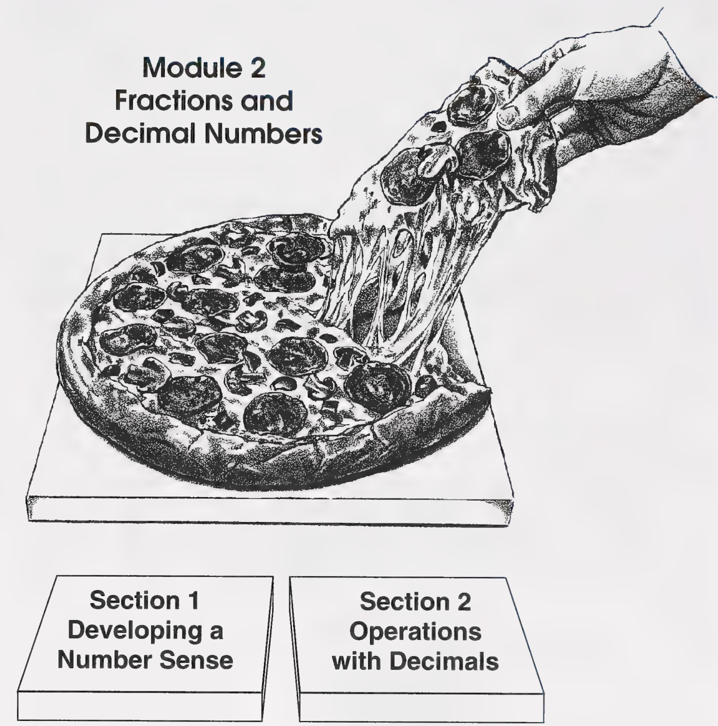
Module Overview

Do you enjoy pizza? Do you like to make your own pizza? Imagine you are planning a party and you will be serving pizza. You are inviting Paula, Louis, Saul, Lisa, Charlene, Kim, and Carl. So, altogether you will be serving eight people. You have two problems: How many pizzas will you make? What kind of pizzas will you make? You decide to make two large pizzas and cut the pizzas in twelfths, so each person will get three-twelfths or one-fourth of a pizza. Paula, Louis, and Saul like ham and pineapple. Lisa likes olives, green peppers, and mushrooms. Charlene, Kim, Carl, and you like pepperoni and bacon. You decide to put ham and pineapple on two-thirds of one pizza and olives, green peppers, and mushrooms on the remaining one-third. The other pizza will be pepperoni and bacon. Will everyone be happy?

As you can see, there are some practical applications to fractions.

In this module you will develop a number sense for fractions. You will work with different forms of fractions: proper fractions, improper fractions, mixed numbers, and decimal numbers. You will add, subtract, multiply, and divide decimal numbers. To do this, you will use paper-and-pencil methods, a calculator, and mental math. You will also practise estimating sums, differences, products, and quotients so you can decide if a calculated answer is reasonable.

Make sure you are well-fed before you begin this module. All this talk of food may make you hungry!



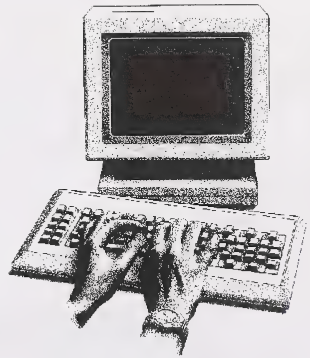
Evaluation

Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete three assignments. The mark distribution is as follows:

Section 1 Assignment	**50 marks**
Section 2 Assignment	**30 marks**
Final Module Assignment	**20 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.

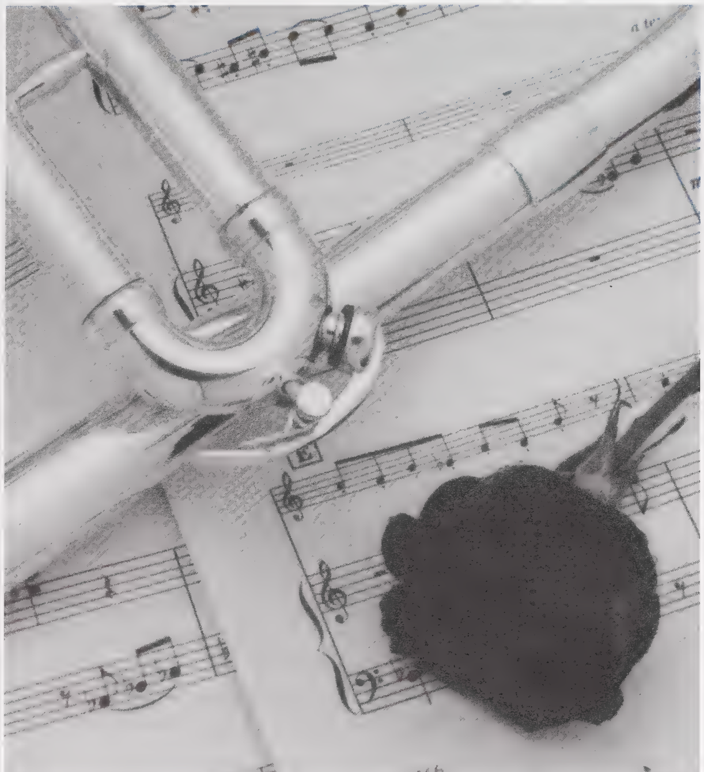
If you are working on a CML terminal, you will have a module test as well as a module assignment.



Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Section 1: Developing a Number Sense



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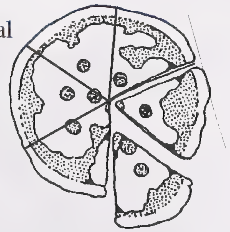
Do you play a musical instrument? Can you read music? How many eighth notes equal a whole note? a half note? a quarter note? If you hold a whole note for four beats, for how many beats do you hold a half note? a quarter note? an eighth note?

Musicians must have a good sense of fractions.

In this section you will develop your number sense for fractions. You will discover that numbers have many different, but equivalent forms. You will write numbers less than one as proper fractions and decimal numbers. You will write fractions greater than one as improper fractions, mixed numbers, and decimal numbers. You will change fractions from one form to another, and you will compare and order fractions and decimal numbers.

Activity 1: Fractions



You have been working with fractions for several years now. You have found that fractions are important because not all quantities can be expressed by whole numbers. For example, six people want to share a pizza so that each person gets an equal portion. Each person will get one-sixth of the pizza.

The fraction one-sixth means $1 \div 6$ or 1 of 6 equal parts.

The fraction may be written as $\frac{1}{6}$.

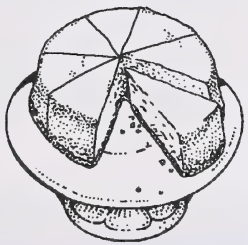
The top number in the fraction shows the number of parts and the bottom number shows the number of equal parts in the whole.

The top number of a fraction is called the dividend, or more commonly, the **numerator**. The bottom number is called the divisor, or more commonly, the **denominator**. The bar that separates the numbers is called the **vinculum**.

$$\text{vinculum} \longrightarrow \frac{1}{6} \begin{array}{l} \longleftarrow \text{numerator} \\ \longleftarrow \text{denominator} \end{array}$$

Example 1

What fraction of this cake was eaten?



Solution

The number of slices eaten was 1.

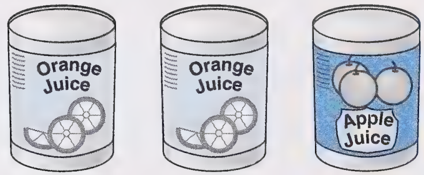
The number of slices in all was 8.

So, $\frac{1}{8}$ of the cake was eaten.

The fraction $\frac{1}{8}$ is read as "1 eighth."

Example 2

What fraction of this set of cans is labelled as orange juice?



Solution

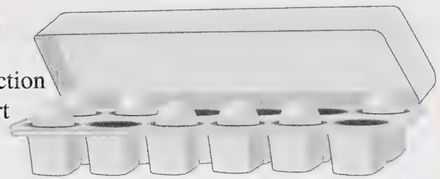
The number of cans labelled as orange juice is 2.

The number of cans in all is 3.

So, $\frac{2}{3}$ of the cans are labelled as orange juice.

The fraction $\frac{2}{3}$ is read as "2 thirds."

1. **a.** There are 12 eggs in a full carton of eggs. Write a fraction to indicate the part of this carton that is filled.

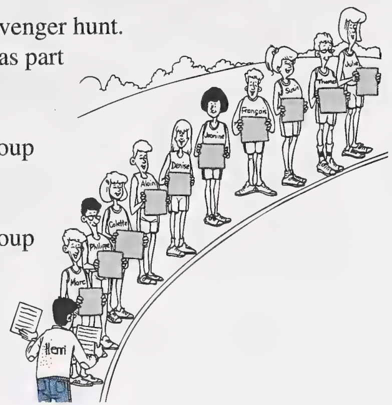


   b. There are 24 cans in a full case. Write a fraction to show the part of this case that is filled.

2. This class is going on a scavenger hunt. **Note:** Consider the leader as part of the class.

   a. What fraction of the group has black hair?

   b. What fraction of the group is wearing jeans?



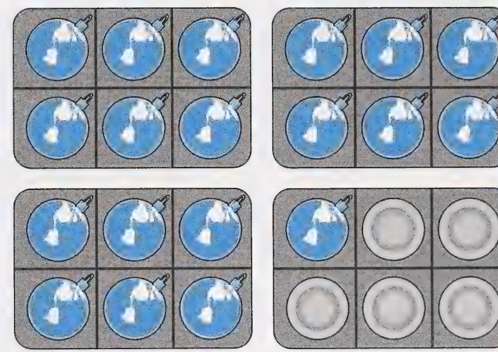
Check your answers by turning to the Appendix.

There are different forms of fractions: **proper fractions**, **improper fractions**, and **mixed numbers**.

A proper fraction is a fraction in which the numerator is less than the denominator. An improper fraction is a fraction in which the numerator is greater than the denominator. A mixed number is a number expressed as a sum of a whole number and a proper fraction.

Example 3

What fraction of the boxes contain ornaments? Express your answer as an improper fraction and as a mixed number.



Solution

Step 1: Find the improper fraction.

The number of sections filled with ornaments is 19.

The number of sections in each box is 6.

So, $\frac{19}{6}$ boxes contain ornaments.

Step 2: Find the mixed number.

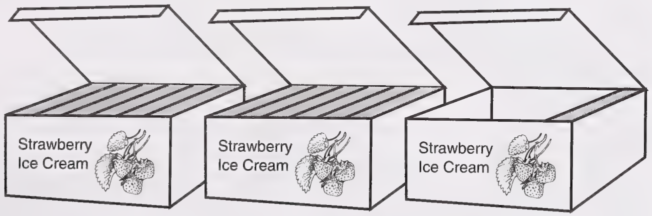
The number of boxes completely filled with ornaments is 3.

There is $\frac{1}{6}$ of another box filled with ornaments.

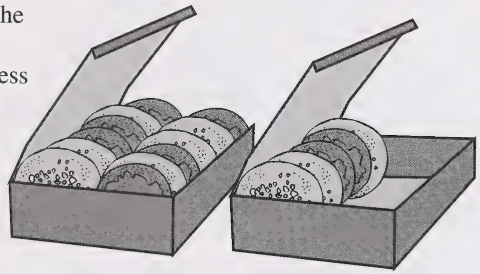
$$3+\frac{1}{6}=3\frac{1}{6}$$

So, $3\frac{1}{6}$ boxes contain ornaments.

3. What fraction of the ice-cream cartons are full? Express your answer as an improper fraction and as a mixed number.



4. What fraction of the cartons contain doughnuts? Express your answer as an improper fraction and as a mixed number.



Check your answers by turning to the Appendix.

Did You Know?

The words *numerator* and *denominator* have Latin origins. Numerator means *numberer*; denominator means *namer*. So, the denominator names the parts and the numerator "numbers" how many parts.

The Babylonians used only fractions with denominators of 60 and 60×60. The fraction $\frac{1}{60}$ was called "the first little part," and the fraction $\frac{1}{60\times 60}$ or $\frac{1}{3600}$ was called "the second little part." These fractions were extensions of the Babylonians' base 60 number system.

The Egyptians wrote all their fractions with a numerator of 1. They used the symbol ⬭ to mean *part*. They used this symbol and other hieroglyphic numerals to write fractions.

III meant 3. ⬭ over III meant $\frac{1}{3}$.

∩II meant 12. ⬭ over ∩II meant $\frac{1}{12}$.

The ancient Greek tradespeople and merchants developed various ways of writing fractions. One way was to put one accent after the numerator and two accents after the denominator. For example, $\gamma'\delta''$ or $\Gamma'\Delta''$ meant $\frac{3}{4}$.

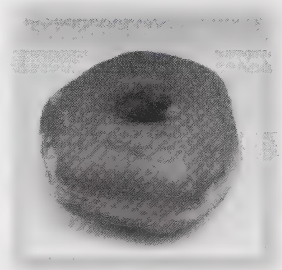
The numerator-over-denominator form of writing fractions was probably first introduced into Europe by the Arabs, who may have gotten the method from the Hindus.

Now Try This

Use a problem-solving strategy to answer the following question.

5. How can you cut a doughnut into eighths with three cuts of a knife?[1]

Note: The pieces must be of equal size.



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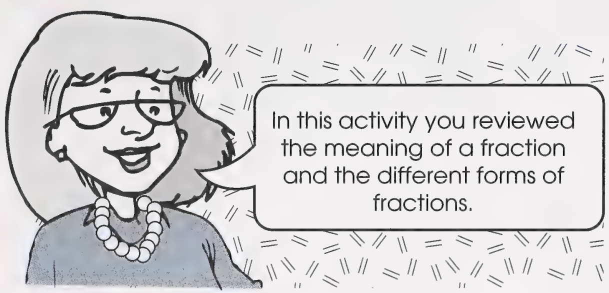
Check your answer by turning to the Appendix.



[1] The National Council of Teachers of Mathematics, Reston, Virginia, for the problem from the *Arithmetic Teacher*, May, 1987.

Activity 2: Decimal Numbers

You have been working with decimal numbers, sometimes called decimal fractions, for several years. Many of the measurements you encounter are expressed as decimal numbers.

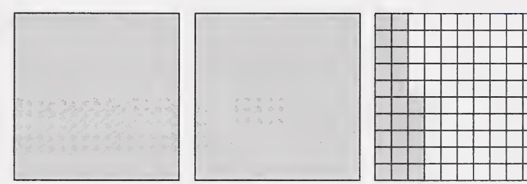
Money and dimensions are two examples. For instance, a birthday card could cost $2.25. It could measure 18.4 cm by 12.2 cm.

The decimal number 2.25 means 2 **and** 25 hundred**ths**.

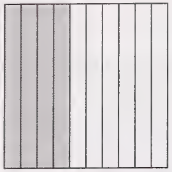
The word ***and*** is used to represent the decimal point.

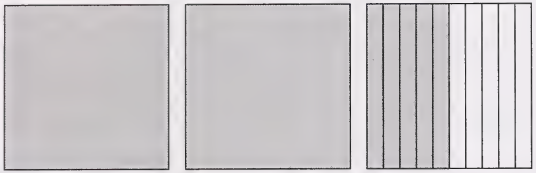
The decimal number 2.25 is another way of expressing the fraction $2\frac{25}{100}$.

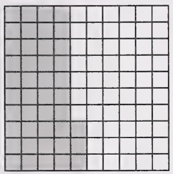
The decimal number 2.25 can be represented by the following diagram.

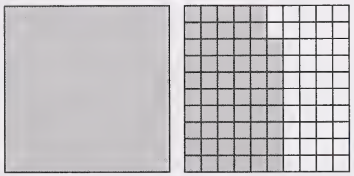


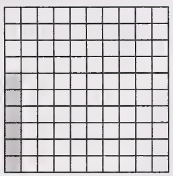
1. What decimal number is represented by each of the following diagrams? Read each number aloud.

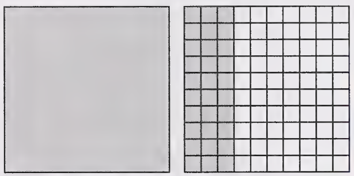
a. 

b. 

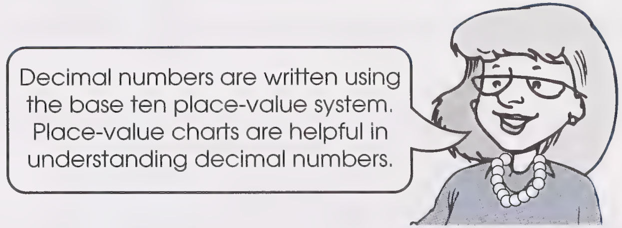
c. 

d. 

e. 

f. 

Check your answers by turning to the Appendix.



Example 1

Write the number 2.47 in a place-value chart.

Solution

or

O	T	H
2 .	4	7

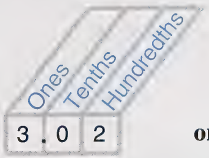
Note: 2.47 is the **standard form** of the number. The **expanded form** of this number is $2 \times 1 + 4 \times 0.1 + 7 \times 0.01$.

The standard form of a number is the usual form of the number. The expanded form of a number is the form expressed as a sum of products; each product shows a digit times its place value.

Example 2

Write the number 3.02 in a place-value chart.

Solution



or

O	T	H
3	. 0	2

Note: 3.02 is the standard form. The expanded form of 3.02 is $3 \times 1 + 0 \times 0.1 + 2 \times 0.01$.

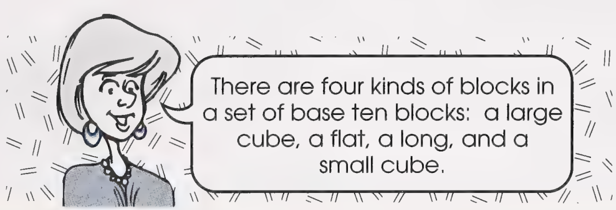
2. Write each of the following numbers in expanded form.

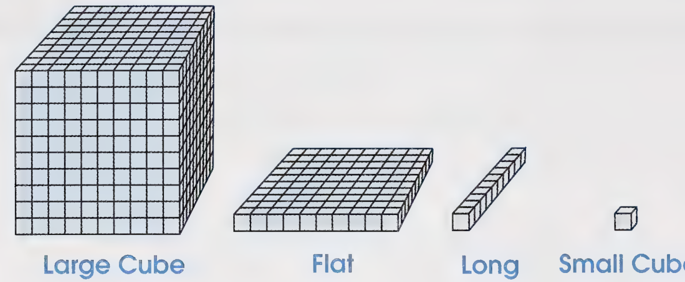
a. 1.25 **b.** 2.07 **c.** 3.40 **d.** 0.57

Check your answers by turning to the Appendix.

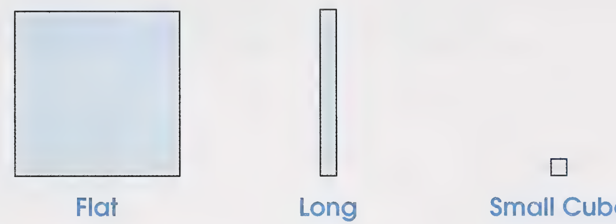
Modelling Decimals

The expanded form of some decimal numbers can be modelled using base ten blocks.

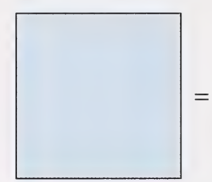




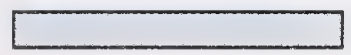
For modelling decimal numbers you will only use flats, longs, and small cubes. The diagrams of flats, longs, and small cubes in this module booklet will show only a top view.



For modelling decimal numbers you will let a flat be one whole.



Because 10 longs equal 1 flat, you will let one long be one-tenth of a whole, or 0.1.

 **= 0.1**

Because 100 small cubes equal 1 flat, you will let a small cube be one-hundredth of a whole, or 0.01.

 **= 0.01**

Example

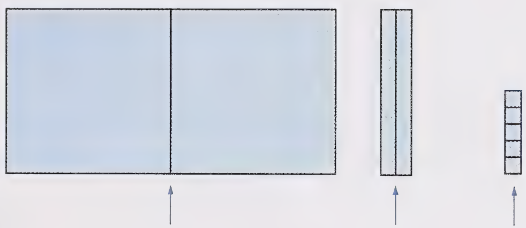
Model 2.25 using base ten blocks.

Solution

Step 1: Write the decimal number in expanded form.

$$2.25 = 2 \times 1 + 2 \times 0.1 + 5 \times 0.01$$

Step 2: Model the decimal number.



You will find cut-out base ten blocks in the Appendix of this module. To make these learning aids easier to use, you can glue photocopies of these pages onto heavier paper, or laminate the learning aids before you cut them out. You may prefer to check with your learning facilitator to see whether your school or school district has the actual base ten blocks available.

3. Use base ten blocks or the cut-out learning aids in the Appendix to model each of the following decimal numbers.

a. 0.3	**b.** 0.48	**c.** 1.07
d. 2.3	**e.** 3.41	**f.** 3.02

Check your answers by turning to the Appendix.

Note: Keep the base ten blocks; you will need them several times in this module.

Expanded Form

So far you have only explored decimal numbers with digits in the ones', tenths', and hundredths' places. Now you will investigate decimal numbers with more places.

Example

Write 0.0028 in expanded form.

Solution

Ones	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths
0	.0	0	2	8	

or

O	T	H	TH	TTH	HTH
0	.0	0	2	8	

$0.0028 = 0 \times 1 + 0 \times 0.1 + 0 \times 0.01 + 2 \times 0.001 + 8 \times 0.0001$

Note: The last place to the right in the decimal 0.0028 is the **ten thousandths**' place. So, the decimal number is read as "28 **ten thousandths**."

4. Write each of the following decimal numbers in expanded form. Read each decimal number aloud.

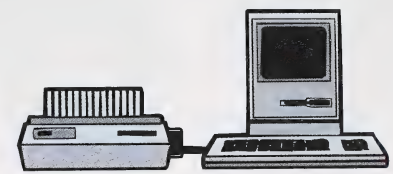
a. 31.035 **b.** 12.0005 **c.** 1.000 36

Check your answers by turning to the Appendix.

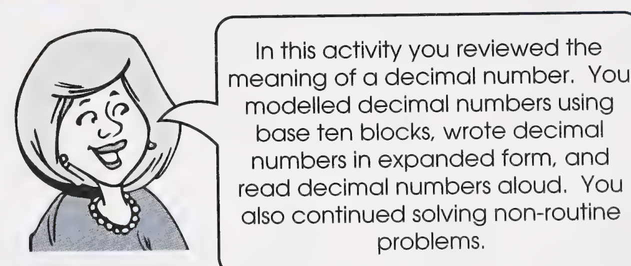
Now Try This

Use a problem-solving strategy to solve the following question.

5. If a computer prints all the whole numbers from 1 to 200 in order, what is the 130th digit which the computer prints?



Check your answer by turning to the Appendix.



Activity 3: Equivalent Fractions and Decimals

Equivalent Fractions

A person often has many different names. For example, a child may call a woman "Mother." A husband may call the same woman "Betty." A nephew may call the same lady "Auntie."

Numbers also have many different names. For example, these fractions name the same part of a whole chocolate bar.

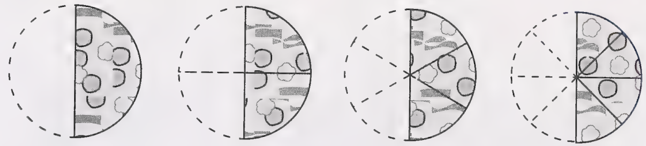
$1 \qquad \frac{2}{2} \qquad \frac{4}{4} \qquad \frac{8}{8}$

From the diagram you can see that there are many equivalent names for 1:

$$1 = \frac{2}{2} = \frac{4}{4} = \frac{8}{8}$$

When fractions name the same part of a whole, they are said to be **equivalent fractions**.

Here is another example of equivalent fractions. These fractions name the same part of a whole pizza.



$\frac{1}{2} \qquad \frac{2}{4} \qquad \frac{3}{6} \qquad \frac{4}{8}$

From the diagram you can see that there are many equivalent names for $\frac{1}{2}$:

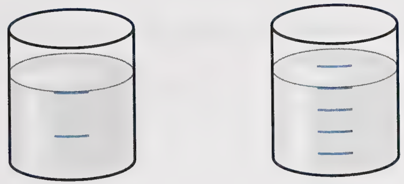
$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$

1. Write the equivalent names for the parts shown in each diagram.

a.

1 = ——

b.



$$\frac{2}{3} \qquad = \qquad \frac{\quad}{\quad}$$

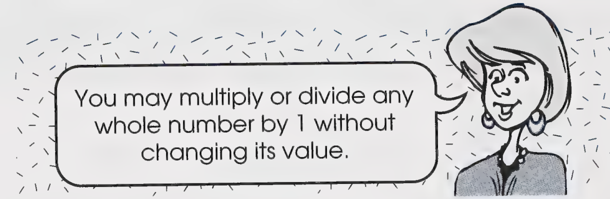
Check your answers by turning to the Appendix.

Pattern blocks may be used to demonstrate equivalent fractions.

You will find cut-out pattern blocks in the Appendix of this module. You may wish to colour these learning aids and glue them to heavy paper.

View the introduction and segment titled "Equivalent Fractions" of the program *Adding and Subtracting Fractions* from the series *Math Moves* and do the video assignment.

Check your answers by viewing the video program.



Here are two examples.

$$2 \times \quad = 2 \qquad 4 \div \quad = 4$$

This property of 1 also applies to fractions. Look at these examples.

$$\frac{3}{4} \times \quad = \frac{3}{4} \qquad \frac{1}{2} \div \quad = \frac{1}{2}$$

This property of 1 helps you write equivalent fractions. Look at these examples.

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4} \qquad \frac{6}{12} \div \frac{3}{3} = \frac{2}{4}$$

Note: There are many different forms of 1.

$$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5}$$

Example 1

Write three equivalent fractions for $\frac{1}{2}$.

Solution

Write equivalent fractions by multiplying the numerator and the denominator by the same number. Multiply by 2, 3, 4,….

$$\frac{1}{2} \underset{\times 2}{\overset{\times 2}{=}} \frac{2}{4} \qquad \frac{1}{2} \underset{\times 3}{\overset{\times 3}{=}} \frac{3}{6} \qquad \frac{1}{2} \underset{\times 4}{\overset{\times 4}{=}} \frac{4}{8}$$

So, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$.

Example 2

Write three equivalent fractions for $\frac{6}{12}$.

Solution

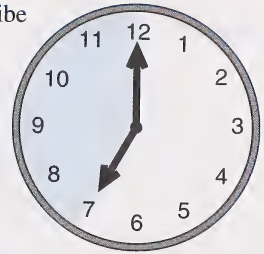
Write equivalent fractions by dividing the numerator and the denominator by the same number.

The numerator 6 and the denominator 12 are each divisible by 2, 3, and 6.

$$\frac{6}{12} \underset{\div 2}{\overset{\div 2}{=}} \frac{3}{6} \qquad \frac{6}{12} \underset{\div 3}{\overset{\div 3}{=}} \frac{2}{4} \qquad \frac{6}{12} \underset{\div 6}{\overset{\div 6}{=}} \frac{1}{2}$$

So, $\frac{6}{12} = \frac{3}{6} = \frac{2}{4} = \frac{1}{2}$.

2. Give four equivalent fractions to describe the part of the clock that is shaded.



Check your answers by turning to the Appendix.

Basic Fractions

A fraction is in **simplest form** when it is written with the smallest possible whole-number denominator. A fraction in simplest form is also called a **basic fraction**.

The following are "yes" examples of basic fractions. These fractions are in their simplest form.

$$\frac{1}{3}, \frac{2}{5}, \frac{7}{8}, \frac{9}{10}$$

The following are "no" examples of basic fractions. These fractions are not in their simplest form.

$$\frac{4}{12}, \frac{6}{15}, \frac{14}{16}, \frac{45}{60}$$

You can use division to simplify a fraction.

Example

Give the basic fraction for $\frac{8}{12}$.

Solution

The basic fraction can be found by dividing the numerator and the denominator by the greatest common factor (GCF) of the two numbers.

$$\frac{8}{12} \overset{\div 4}{=} \frac{2}{3}$$

The GCF of 12 and 8 is 4.

The smallest possible whole-number numerator of $\frac{8}{12}$ is 2. The smallest whole-number denominator of $\frac{8}{12}$ is 3.

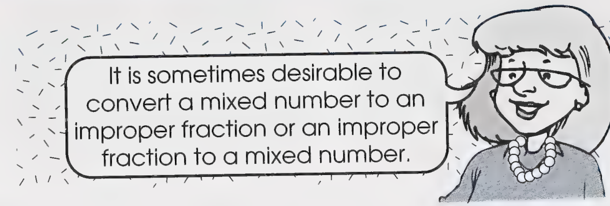
So, the simplest form of $\frac{8}{12}$ is $\frac{2}{3}$. Therefore, $\frac{2}{3}$ is a basic fraction.

3. Write each of the following fractions in simplest form.

a. $\frac{9}{12}$ **b.** $\frac{4}{6}$ **c.** $\frac{24}{96}$ **d.** $\frac{72}{30}$ **e.** $\frac{30}{24}$ **f.** $\frac{45}{40}$

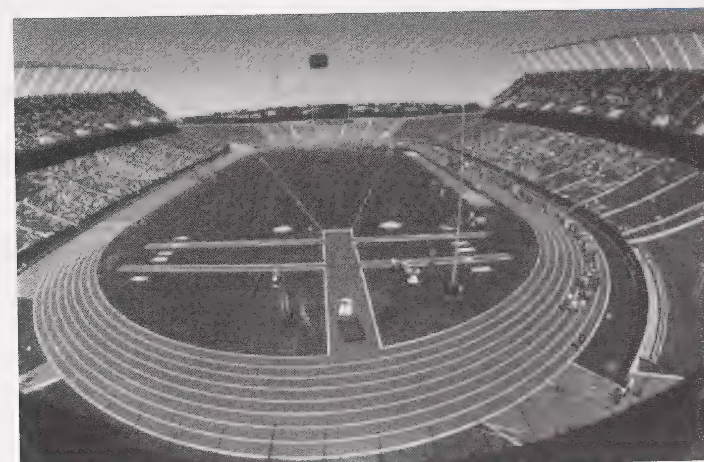
Check your answers by turning to the Appendix.

Converting Between Mixed Numbers and Improper Fractions



Example 1

Seth ran $4\frac{1}{3}$ laps around the track. Express $4\frac{1}{3}$ as an improper fraction.



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Solution

Method 1

$$4\frac{1}{3} = 4 + \frac{1}{3}$$
$$= 1 + 1 + 1 + 1 + \frac{1}{3}$$
$$= \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{1}{3}$$
$$= \frac{12}{3} + \frac{1}{3}$$
$$= \frac{13}{3}$$

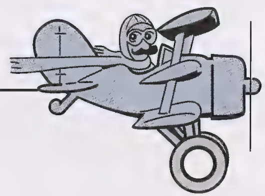
3 thirds + 3 thirds + 3 thirds + 3 thirds = 12 thirds.

So, Seth ran $\frac{13}{3}$ laps of the track.

Method 2

Here is a quick way to find the improper fraction.

- Multiply the denominator of the fraction by the whole number. Then add the numerator of the fraction part to this product.
- Write the resulting number over the denominator of the original fraction.



$$4\frac{1}{3} = \frac{\overset{\text{whole number}}{4} \times \overset{\text{denominator of fraction}}{3} + \overset{\text{numerator of fraction}}{1}}{\underset{\text{denominator of fraction}}{3}}$$
$$= \frac{12 + 1}{3}$$
$$= \frac{13}{3}$$

These steps can be done mentally.

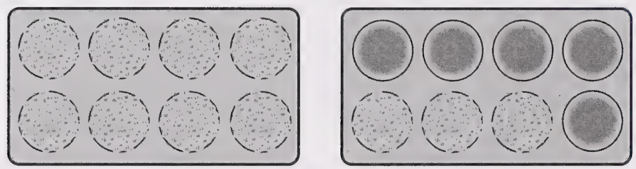
So, Seth ran $\frac{13}{3}$ laps of the track.

4. Express each of these mixed numbers as an improper fraction.

a. $5\frac{1}{4}$ **b.** $3\frac{2}{5}$ **c.** $2\frac{1}{3}$ **d.** $4\frac{3}{5}$

Example 2

There are $\frac{11}{8}$ tins of muffins. Express $\frac{11}{8}$ as a mixed number.



Solution

Method 1

$$\frac{11}{8} = \frac{8}{8} + \frac{3}{8}$$
$$= \frac{8}{8} + \frac{3}{8}$$
$$= 1 + \frac{3}{8}$$
$$= 1\frac{3}{8}$$

11 eighths =
8 eighths + 3 eighths

So, there are $1\frac{3}{8}$ tins of muffins.

Method 2

Here is a quick way to find the mixed number.

- Divide the numerator of the improper fraction by the denominator.
- Write the whole number part of the quotient.
- Then divide the remainder by the denominator and write this part of the quotient as a fraction.

$$\begin{array}{r} 1 \\ 8\overline{)11} \\ \underline{8} \\ 3 \end{array}$$

denominator → 8; 1 ← whole number part of quotient; 11 ← numerator; 3 ← remainder

The remainder is
divided by the
denominator too.

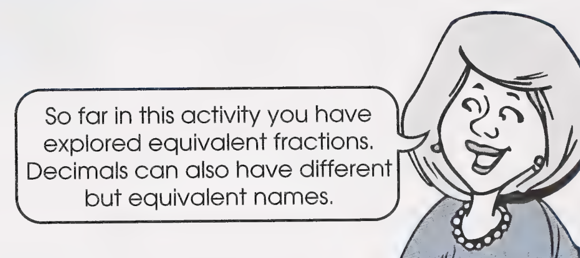
$$\frac{11}{8} = \text{whole number part of quotient} + \frac{\text{remainder}}{\text{denominator}}$$
$$= 1 + \frac{3}{8}$$
$$= 1\frac{3}{8}$$

So, there are $1\frac{3}{8}$ tins of muffins.

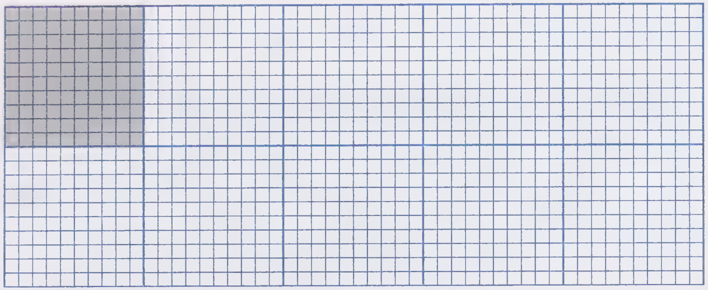
5. Express each of these improper fractions as a mixed number.

a. $\frac{10}{3}$ **b.** $\frac{16}{5}$ **c.** $\frac{21}{4}$ **d.** $\frac{33}{2}$

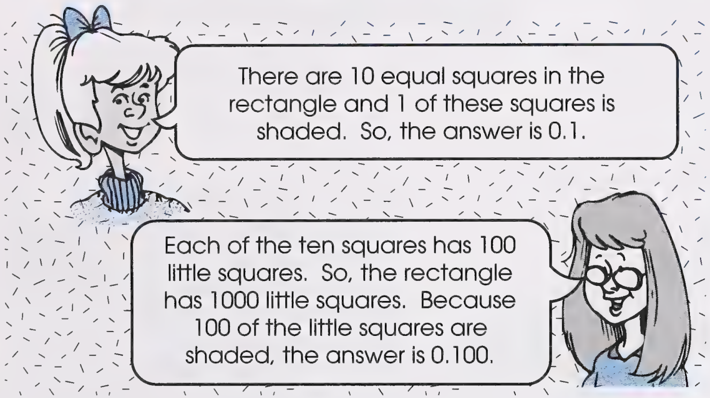
Equivalent Decimals



For example, what decimal fraction of this rectangle is shaded?



The students have different answers.



Both students are correct; 0.1 and 0.100 are **equivalent decimals**.

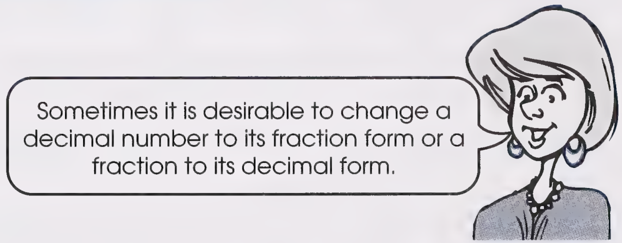
When decimals name the same part of a whole, they are said to be equivalent decimals. It is important to remember that zeros at the end of a decimal number do not change its value.

6. Write two equivalent decimals for each of the following decimal numbers.

a. 0.60 **b.** 0.7 **c.** 1.280 **d.** 19.05

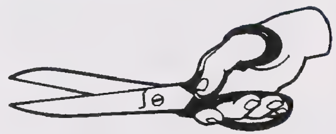
Check your answers by turning to the Appendix.

Converting Between Decimal Numbers and Fractions



Example 1

Fred took 0.25 h to cut out the pieces of material for his apron. Express 0.25 as a fraction in basic form.



Solution

Step 1: Use a place-value chart to help you make the conversion.

Ones	Tenths	Hundredths
0 .	2	5

or

O	T	H
0 .	2	5

The decimal number 0.25 means 25 hundredths or $\frac{25}{100}$.

Step 2: Find the basic fraction.

$$\frac{25}{100} \underset{\div 25}{\overset{\div 25}{=}} \frac{1}{4}$$

The GCF of 25 and 100 is 25.

So, Fred took $\frac{1}{4}$ h to cut out the pieces of material for his apron.

7. Express each of these decimal numbers as a fraction in simplest form.

a. 0.3 **b.** 0.26 **c.** 0.05
d. 4.25 **e.** 2.875 **f.** 3.036

Check your answers by turning to the Appendix.

Example 2

The birds at Gordie's house eat $\frac{1}{8}$ of a bag of seeds each week. Express this fraction as a decimal number.

Solution

Step 1: Write an equivalent fraction with a denominator of 10, 100, 1000,… .

The three consecutive dots (ellipsis) indicate the pattern continues.

$$\frac{1}{8} \underset{\times 125}{\overset{\times 125}{=}} \frac{125}{1000}$$

Step 2: Write the equivalent fraction as a decimal number.

$$\frac{125}{1000} = 0.125$$

Both $\frac{125}{1000}$ and 0.125 are read as "125 thousandths."

So, the birds eat 0.125 bags of seed every week.

8. Express each of these fractions as a decimal number.

a. $\frac{3}{4}$ b. $\frac{2}{5}$ c. $\frac{5}{8}$ d. $1\frac{3}{10}$ e. $2\frac{1}{2}$

Check your answers by turning to the Appendix.

You can use division to change a fraction to a decimal number.

Example 3

A store takes $\frac{1}{8}$ off the price of its merchandise. Express the fraction as a decimal number.

Solution

Method 1: Using Long or Short Division

Step 1: Divide the numerator by the denominator. You may use long division or short division. Keep dividing until there is a remainder of zero or the remainders start to repeat.

$\frac{1}{8}$ can mean $1 \div 8$.

$$\begin{array}{r} 0.125 \\ 8\overline{)1.000} \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

← There is a remainder of zero.

$$8\overline{)1.0_2 0_4 0_0}\quad \text{quotient } 0.1\ 2\ 5$$

There is a remainder of zero.

Step 2: Write the fraction as a decimal number.

$$\frac{1}{8} = 0.125$$

The store takes 0.125 off the price of its merchandise.

When you divide the numerator of a fraction by the denominator and there is a remainder of zero, the resulting decimal number is called a **terminating decimal**.

Method 2: Using a Calculator

You may also use a calculator to change a fraction to a decimal number.

1 ÷ 8 =

0.125

The store takes 0.125 off the price of its merchandise.

Example 4

Lucy hit the baseball safely $\frac{5}{11}$ of the times that she was up to bat. Express the fraction as a decimal number.

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Solution

Method 1: Using Long or Short Division

Step 1: Divide the numerator by the denominator. You may use long division or short division. Keep dividing until there is a remainder of zero or the remainders start to repeat.

$\frac{5}{11}$ can mean $5 \div 11$.

$$\begin{array}{r} 0.454 \\ 11\overline{)5.000} \\ \underline{44} \\ 60 \\ \underline{55} \\ 50 \\ \underline{44} \\ 6 \end{array}$$

The remainder 6 occurred before.

$$11\overline{)5.0_6 0_5 0_6}\quad 0.454$$

The remainder 6 occurred before.

Step 2: Write the fraction as a decimal number.

$$\frac{5}{11} = 0.\overline{45}$$

So, Lucy hit safely $0.\overline{45}$ of the times she was at bat.

When you divide the numerator by the denominator and the remainders start to repeat, the resulting decimal number is a **repeating decimal number**.

A repeating decimal number has an infinite number of digits with a repeating pattern.

$$\frac{5}{11} = 0.4545\ldots$$

The three dots (ellipsis) indicate the pattern continues.

Sometimes a repeating decimal number is written with dots over the first number and last number of the repeating block of digits.

$$\frac{5}{11} = 0.\dot{4}\dot{5}$$

Usually, however, a repeating decimal number is written with a bar placed over the repeating block of digits.

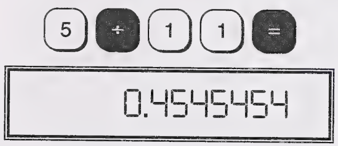
$$\frac{5}{11} = 0.\overline{45}$$

The bar over the repeating block of digits may be called the **vinculum**.

You will recall the bar that separates the numerator and the denominator of a fraction is also called the vinculum.

Method 2: Using a Calculator

You may also use a calculator to change a fraction to a decimal number.



So, Lucy hit safely $0.\overline{45}$ of the times she was at bat.

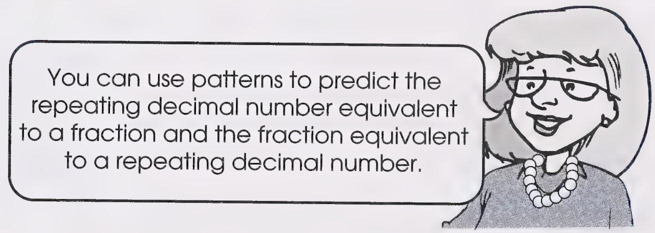
Note: Calculators differ in the number of digits they will display. (The one shown in the example displays a maximum of 8 digits.) Some calculators round off a decimal number if all the digits cannot be displayed; other calculators do not round off the decimal number. (The one in the example does not round off.)

9. Write each of the following fractions as a decimal number. If the fraction is equivalent to a repeating decimal, use the bar notation to clearly show the repeating block of digits.

a. $\frac{7}{9}$ **b.** $\frac{5}{12}$ **c.** $\frac{9}{16}$

d. $\frac{9}{11}$ **e.** $\frac{3}{40}$ **f.** $\frac{2}{13}$

Check your answers by turning to the Appendix.



10. Use this series of fractions to answer the following set of questions.

$$\frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \frac{4}{9}$$

a. Convert each of the fractions in the series to a repeating decimal number.

b. What pattern do you see?

c. Use the pattern to predict a repeating decimal number equivalent to each fraction in this series.

$$\frac{5}{9}, \frac{6}{9}, \frac{7}{9}, \frac{8}{9}$$

11. Use this series of fractions to answer the following set of questions.

$$\frac{1}{99}, \frac{2}{99}, \frac{3}{99}, \frac{4}{99}, \frac{5}{99}$$

a. Convert each of the fractions in the series to a repeating decimal number.

b. What pattern do you see?

c. Use the pattern to predict a repeating decimal number equivalent to each fraction in this series.

$$\frac{13}{99}, \frac{23}{99}, \frac{47}{99}, \frac{68}{99}, \frac{94}{99}$$

12. Use the patterns you discovered in questions 10 and 11 to find a fraction equivalent to each of the following repeating decimal numbers.

a. $2.\overline{7}$ **b.** $0.\overline{14}$ **c.** $3.\overline{26}$ **d.** $1.\overline{8}$

Check your answers by turning to the Appendix.

Did You Know?

The decimal number form is used in the metric system of measurement because metric measurement is based on divisions of ten. For example, a lobster can weigh 1.5 kg and a person with a fever could have a temperature of 38.7°C.

The fraction form of numbers is often used in non-metric systems of measurement. For example, it can take $1\frac{3}{4}$ h to clean a car and a diamond can be $1\frac{1}{2}$ carats in size.

In Quebec, a comma is used as a decimal marker. For example, the terminating decimal number 3.754 is written as 3,754. In the other provinces, a period is used as a decimal marker.

Problem Solving

Many problems involve dividing two whole numbers. The way the answer is expressed depends on the situation. The answer could be a fraction, a mixed number, or a decimal number. However, in some situations it is more reasonable to round off the answer to a whole number. The answer could even be the remainder in some situations.

Example 1

Charlie's marching band rehearsed a total of 29 hours in the last 4 days before the parade. If they spaced their rehearsals equally over the 4 days, how many hours did they practise each day?

Solution

$$4\overline{)29} = 7\text{R}1 = 7\tfrac{1}{4}$$
$$\begin{array}{r} 29 \\ \underline{28} \\ 1 \end{array}$$

$$4\overline{)29.00} = 7.25$$
$$\begin{array}{r} 29.00 \\ \underline{28} \\ 1\,0 \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

2 9 ÷ 4 =

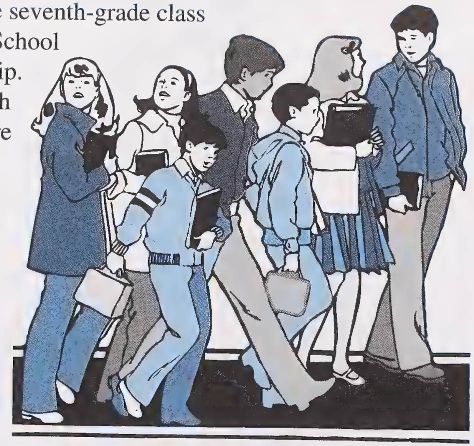
7.25

The band practised $7\frac{1}{4}$ h each day.

The band practised 7.25 h each day.

Example 2

The 29 students in the seventh-grade class at the Willow Creek School are going on a field trip. If 4 students fit in each car, how many cars are needed?



Solution

$$4\overline{)29} = 7\text{R}1 = 7\tfrac{1}{4}$$
$$\begin{array}{r} 29 \\ \underline{28} \\ 1 \end{array}$$

$$4\overline{)29.00} = 7.25$$
$$\begin{array}{r} 29.00 \\ \underline{28} \\ 1\,0 \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

2 9 ÷ 4 =

7.25

The students will need 8 cars.

Note: It is not reasonable to have $7\frac{1}{4}$ cars or 7.25 cars. The answer must be rounded up to 8 cars because 7 cars would be too few.

Example 3

Annette has collected 29 cassette tapes. She wants to arrange them in a box which will hold 4 tapes in each row. How many tapes will she put in the last row?

Solution

Find the remainder. (If you use long division, the remainder is easily found. If you use a calculator, the answer will be displayed as a decimal number; you can find the remainder by subtracting the whole number part and then multiplying the decimal number part by the divisor.)

$$\begin{array}{r} 7 \\ 4\overline{)29} \\ \underline{28} \\ 1 \end{array}$$

This is the remainder.

2 9 ÷ 4 =   Divide.

7.25

− 7 =   Subtract the whole number part.

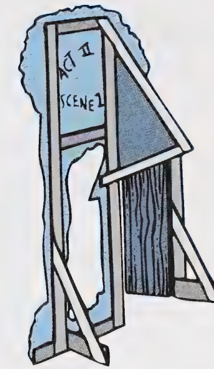
0.25

× 4 =   Multiply by the divisor.

1.0 ← This is the remainder.

There will be 1 tape in the last row.

13. A theatre group sold 36 tickets for a total of $198. What was the price of one ticket?

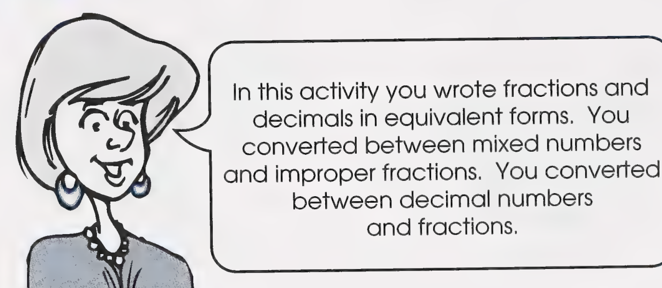


14. The Egg Marketing Board receives a shipment of 4000 eggs. How many dozen eggs do they receive?

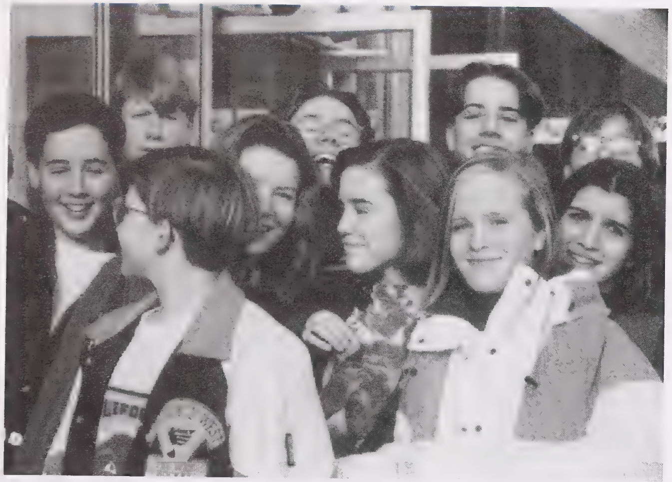
15. The teen club had a bottle drive to raise money. They collected 1335 bottles. They put the bottles in boxes with 24 bottles to a box.

a. How many boxes were needed?
b. How many boxes were full?
c. How many bottles were in the partly filled box?

Check your answers by turning to the Appendix.



Activity 4: Comparing and Ordering Fractions and Decimals



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These friends all have different heights. If you lined up the friends in order of height from shortest to tallest, who would be first? Who would be last?

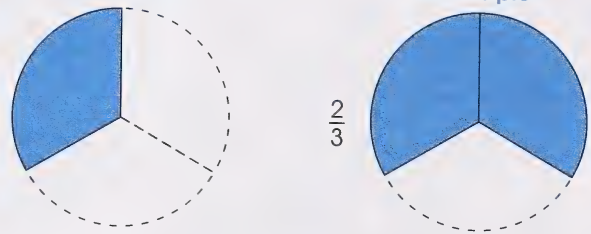
In this activity you will compare and order fractions and decimal numbers. You will use these symbols:

= is equal to   < is less than   > is greater than

You will begin by comparing fractions with like denominators.

Example 1

Adam has $\frac{1}{3}$ of a pie. Anna has $\frac{2}{3}$ of a pie. Who has less pie?

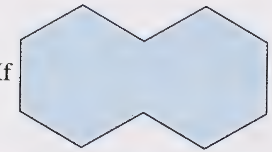


Solution

From the diagram, you can see that $\frac{1}{3}$ is less than $\frac{2}{3}$.

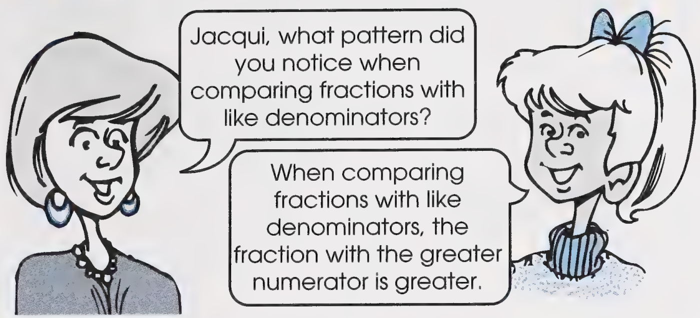
$\frac{1}{3} < \frac{2}{3}$ The inequality sign points to the number that is less.

Adam has less pie.

1. If  = 1, model each pair of fractions using pattern blocks. Then use < or > to compare the fractions.

a. $\frac{1}{6}$ ◯ $\frac{5}{6}$ b. $\frac{3}{4}$ ◯ $\frac{1}{4}$ c. $\frac{5}{12}$ ◯ $\frac{7}{12}$

Check your answers by turning to the Appendix.



Use the pattern Jacqui observed to answer question 2 mentally.

2. Use < or > to show which fraction is the greater of the two.

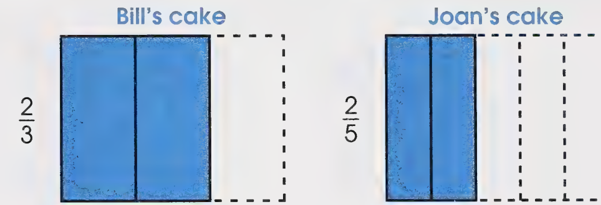
a. $\frac{5}{8}$ ◯ $\frac{3}{8}$ b. $\frac{9}{7}$ ◯ $\frac{11}{7}$ c. $\frac{7}{10}$ ◯ $\frac{9}{10}$

Check your answers by turning to the Appendix.

Now you will compare fractions with like numerators.

Example 2

Bill has $\frac{2}{3}$ of a cake. Joan has $\frac{2}{5}$ of a cake. Who has more cake?

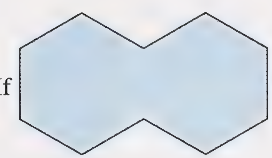


Solution

From the diagram, you can see that $\frac{2}{3}$ is greater than $\frac{2}{5}$.

$\frac{2}{3} > \frac{2}{5}$ The inequality sign points to the number that is less.

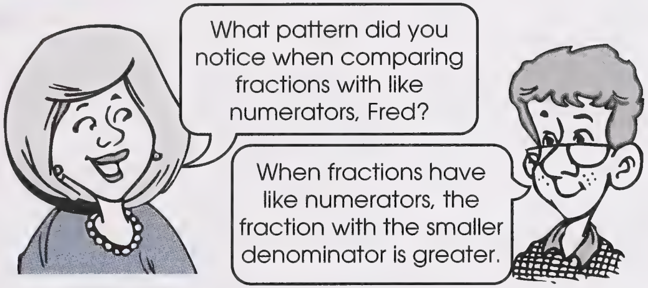
Bill has more cake.

3. If  = 1, model each pair of fractions using pattern blocks. Then use < or > to compare the fractions.

a. $\frac{1}{6}$ ◯ $\frac{1}{2}$ b. $\frac{1}{2}$ ◯ $\frac{1}{3}$

c. $\frac{5}{6}$ $\frac{5}{12}$ **d.** $\frac{7}{4}$ $\frac{7}{12}$

Check your answers by turning to the Appendix.



Use the pattern Fred observed to answer question 4 mentally.

4. Use < or > to show which fraction is the greater of the two.

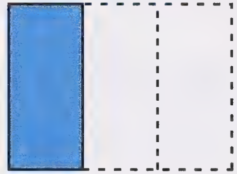
a. $\frac{4}{11}$ $\frac{4}{5}$ **b.** $\frac{3}{8}$ $\frac{3}{10}$ **c.** $\frac{7}{2}$ $\frac{7}{3}$

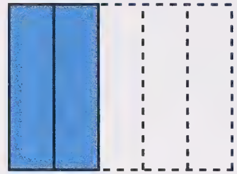
Check your answers by turning to the Appendix.

Now you will compare fractions with both different numerators and different denominators.

Example 3

Belinda has $\frac{1}{3}$ of a chocolate bar. Bill has $\frac{2}{5}$ of a chocolate bar. Who has more chocolate?

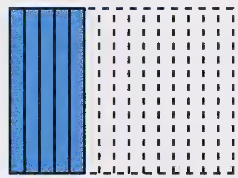




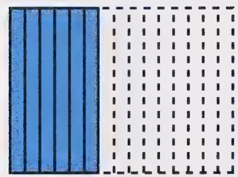
Solution

It is difficult to tell which person has more chocolate simply by looking at the diagram.

One way to make it easier to compare the amounts is to cut both chocolate bars into pieces that are the same size. By doing this you would be renaming the fractions with common denominators.



$\frac{1}{3} = \frac{5}{15}$



$\frac{2}{5} = \frac{6}{15}$

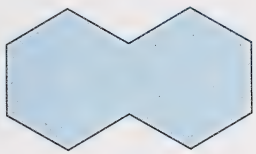
From the new diagram, you can see that $\frac{6}{15}$ is greater than $\frac{5}{15}$.

$$\frac{6}{15} > \frac{5}{15}$$

$$\therefore \ \frac{2}{5} > \frac{1}{3}$$

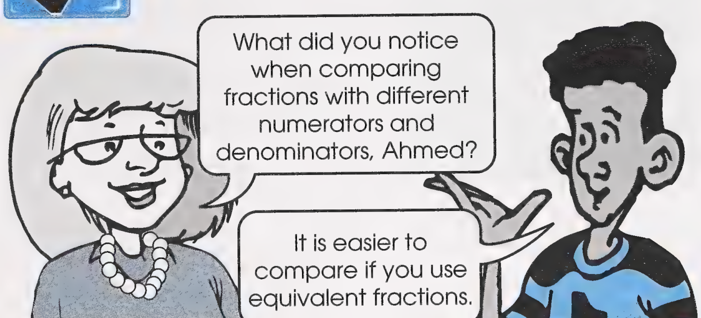
The symbol ∴ is read as "so" or "therefore."

Bill has more chocolate.

5. If  = 1, model each pair of fractions using pattern blocks. Then use < or > to compare the fractions.

a. $\frac{1}{2}$ ◯ $\frac{7}{12}$ **b.** $\frac{3}{4}$ ◯ $\frac{5}{6}$ **c.** $\frac{11}{12}$ ◯ $\frac{3}{4}$

Check your answers by turning to the Appendix.



You do not need objects or diagrams to compare fractions having different numerators or denominators.

Example 4

Which is greater, $\frac{2}{3}$ or $\frac{4}{7}$?

Solution

Step 1: Find equivalent fractions with a common denominator.

$$\frac{2 \times 7}{3 \times 7} = \frac{14}{21} \qquad \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$$

Step 2: To decide which fraction is greater, compare the fractions with the common denominators.

$$\frac{14}{21} > \frac{12}{21}$$

$$\therefore \ \frac{2}{3} > \frac{4}{7}$$

6. Use < or > to show which fraction is the greater of the two.

a. $\frac{2}{5}$ ◯ $\frac{3}{7}$ **b.** $\frac{7}{9}$ ◯ $\frac{2}{3}$

c. $\frac{3}{4}$ ◯ $\frac{4}{5}$ **d.** $\frac{5}{6}$ ◯ $\frac{11}{12}$

7. If it took Marnee $\frac{1}{2}$ h to complete one activity of the math course and Darwin did the activity in $\frac{3}{4}$ h, which person took more time to complete the work?

h = hour

8. Sue is a diamond expert. Sue bought a $\frac{1}{2}$-carat diamond, a $\frac{5}{8}$-carat diamond, and a $\frac{2}{3}$-carat diamond. Which is the biggest diamond of the three?

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Check your answers by turning to the Appendix.

When comparing fractions, the numerators and denominators are important. When comparing decimals, place value is important.

Example 5

Which is greater, 0.35 or 0.4?

Solution

Line up the digits in each place value. Then, compare the digits from left to right.

0 . 3 5

same different

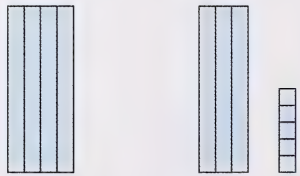
0 . 4

Both 0.35 and 0.4 have no ones, but 0.4 has 4 tenths and 0.35 has only 3 tenths.

4 tenths > 3 tenths

$\therefore$ 0.4 > 0.35

Note: A model may help you visualize this.



9. Use < or > to show which decimal is the greater of the two.

a. 8.15 8.05 **b.** 1.03 1.02

c. 0.7 0.07 **d.** 5.321 4.321

e. 0.06 ◯ 0.1 **f.** 0.020 ◯ 0.3

Check your answers by turning to the Appendix.

Finding equivalent fractions or decimal numbers is helpful when comparing numbers in fraction and decimal form.

Example 6

Which is greater, 0.4 or $\frac{1}{2}$?

Solution

Method 1: Finding Equivalent Fractions

Step 1: Find equivalent fractions with a common denominator.

$$0.4 = \frac{4}{10} \qquad \frac{1}{2} = \frac{5}{10}$$

Step 2: To decide which is greater, compare the fractions with common denominators.

$$\frac{5}{10} > \frac{4}{10}$$

$$\therefore \frac{1}{2} > 0.4$$

Method 2: Finding Equivalent Decimal Numbers

Step 1: Find equivalent decimal numbers.

$$0.4 \qquad \frac{1}{2} = 0.5$$

Step 2: To decide which is greater, compare the decimal numbers.

$$0.5 > 0.4$$

$$\therefore \frac{1}{2} > 0.4$$

10. Use < or > to show which number is the greater of the two.

a. $\frac{7}{8}$ ◯ 0.75 **b.** $\frac{5}{8}$ ◯ 0.6

c. $\frac{2}{3}$ ◯ 0.6 **d.** $\frac{1}{7}$ ◯ 0.2

Check your answers by turning to the Appendix.

You are now ready to order fractions and decimal numbers.

Listing numbers in order of size is called ordering the numbers.

When you have a set of numbers you may list the numbers in **ascending order** or **descending order**. Ascending order is from least to greatest. Descending order is from greatest to least. You can think of stairs to help you remember these words.

To descend a staircase means to go down. To ascend the staircase means to go up.

Example 7

Sandy, Marc, and Ahmed went to a bakery. Sandy bought $\frac{1}{2}$ dozen brownies. Marc bought $\frac{5}{12}$ of a dozen brownies. Ahmed bought $\frac{5}{6}$ of a dozen brownies. List the purchases in ascending order.

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Solution

Step 1: Write each fraction with a common denominator.

Sandy's purchase		Marc's purchase		Ahmed's purchase
$\frac{1}{2} = \frac{6}{12}$	and	$\frac{5}{12}$	and	$\frac{5}{6} = \frac{10}{12}$

Step 2: Order the numerator numbers from least to greatest.

$$\frac{5}{12} < \frac{6}{12} < \frac{10}{12}$$

$$\therefore \frac{5}{12} < \frac{1}{2} < \frac{5}{6}$$

Each inequality sign points to the smaller number.

So, the purchases in ascending order are Marc's $\frac{5}{12}$ dozen, Sandy's $\frac{1}{2}$ dozen, and Ahmed's $\frac{5}{6}$ dozen.

11. Write each of the following sets of numbers in descending order.

a. $\left\{\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{8}\right\}$

b. $\left\{\frac{3}{8}, 0.4, \frac{1}{2}, 0.75\right\}$

Check your answers by turning to the Appendix.

Now Try This

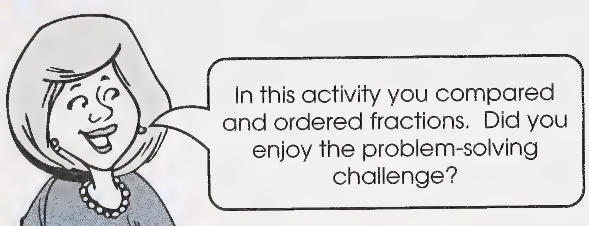
Use the Internet to find information on math terms you may not know or that you are having trouble understanding. This is the uniform resource locator (URL) for the On-line Mathematics Dictionary:

http://www.mathpro.com/math/glossary/glossary.html

Use a problem-solving strategy to answer the following question.

12. At a track meet, the competitors received 5 points for each first-place ribbon and 3 points for each second-place ribbon. Jacinda received 12 points. What ribbons did she win?

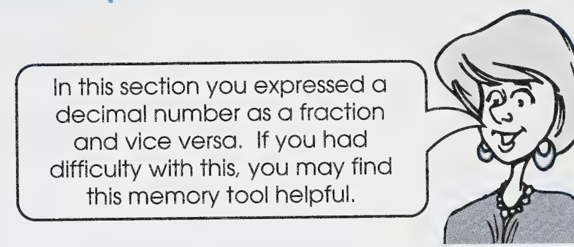
Check your answer by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help



Example 1

Express 0.1875 as a fraction. Do not simplify the fraction.

Solution

Use the relationship between the number of decimal places and the number of zeros in the denominator to write an equivalent fraction.

$$\underbrace{0.\,1875}_{\text{four decimal places}} = \frac{1875}{\underbrace{1\,0\,000}_{\text{four zeros}}}$$

Example 2

Express $\frac{3}{8}$ as a decimal number.

Solution

Write an equivalent fraction with a denominator of 10, 100, 1000,… . Then use the relationship between the number of zeros in the denominator and the number of decimal places to write an equivalent decimal number.

$$\frac{3}{8} = \frac{375}{1\ 000} \longleftarrow \text{three zeros}$$
$$= 0.375 \longleftarrow \text{three decimal places}$$

1. Write each of the following decimal numbers as a fraction. Do not simplify the fraction.

   a. 0.01 **b.** 0.239 **c.** 5.14
   d. 6.005 **e.** 9.32 **f.** 0.037

2. Write each of the following as a decimal number.

   a. $\frac{7}{8}$ **b.** $\frac{21}{25}$ **c.** $1\frac{3}{8}$
   d. $2\frac{7}{125}$ **e.** $\frac{39}{40}$ **f.** $3\frac{3}{16}$

Check your answers by turning to the Appendix.

Here is a quick way to find the basic fraction.

- Write the numerator and the denominator as a product of prime factors.
- Cancel the common factors.
- Multiply the remaining factors.

Example 3

Write $\frac{8}{12}$ in simplest form.

Solution

$$\frac{8}{12} = \frac{2 \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times 3}$$
$$= \frac{2}{3}$$

Line up the common prime factors in the numerator and the denominator.

Example 4

Write $\frac{18}{24}$ in simplest form.

Solution

$$\frac{18}{24}=\frac{\overset{1}{\cancel{2}}\times\overset{1}{\cancel{3}}\times 3}{\underset{1}{\cancel{2}}\times 2\times 2\times\underset{1}{\cancel{3}}}$$

$$=\frac{3}{4}$$

Line up the common prime factors in the numerator and the denominator.

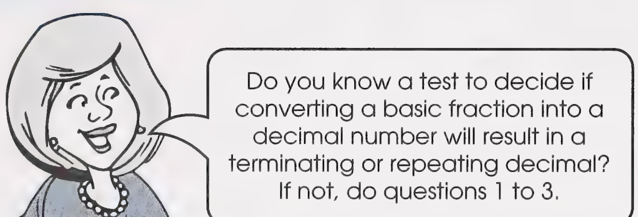
3. Write the basic fraction for each of the following.

a. $\frac{4}{10}$ b. $\frac{6}{15}$ c. $\frac{27}{15}$

d. $\frac{32}{24}$ e. $\frac{16}{36}$ f. $\frac{18}{42}$

Check your answers by turning to the Appendix.

Enrichment



1. Copy and complete a table like the following.

Fraction	Fraction with the Denominator Expressed As a Product of Prime Factors	Equivalent Decimal Number	Type of Decimal Number
$\frac{7}{8}$	$\frac{7}{2\times 2\times 2}$	0.875	terminating
$\frac{3}{20}$			
$\frac{4}{5}$			
$\frac{1}{50}$			
$\frac{9}{11}$			
$\frac{7}{12}$	$\frac{7}{2\times 2\times 3}$	$0.58\overline{3}$	repeating
$\frac{5}{18}$			
$\frac{5}{13}$			
$\frac{1}{7}$			

2. a. Compare the prime factors in the denominators of the basic fractions equivalent to terminating decimals in question 1. What do you notice?

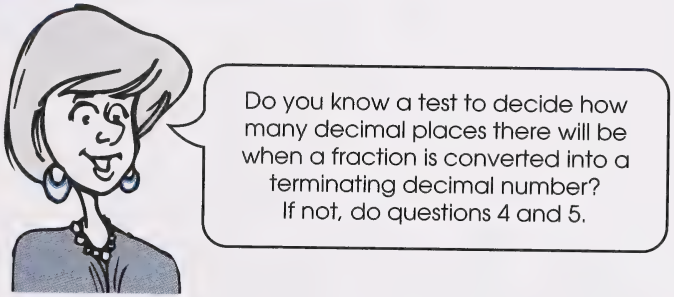
   b. Compare the prime factors in the denominators of the basic fractions equivalent to repeating decimals in question 1.

3. Use the pattern you discovered in question 2 to decide if each of the following basic fractions will convert to a terminating decimal number or a repeating decimal number.

a. $\frac{11}{12}$ b. $\frac{1}{16}$ c. $\frac{10}{49}$

d. $\frac{9}{40}$ e. $\frac{8}{125}$ f. $\frac{2}{81}$

Check your answers by turning to the Appendix.

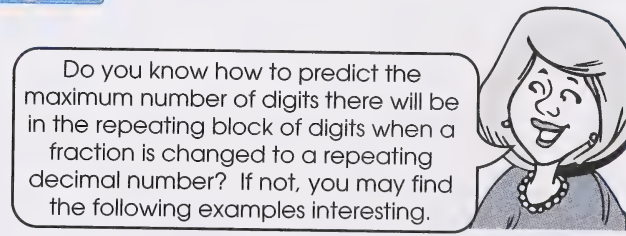


4. Look at the prime factors in the denominators of the basic fractions equivalent to terminating decimals in question 1. Compare the number of 2s and 5s to the number of decimal places. What do you notice?

5. Use the pattern you discovered in question 4 to decide how many decimal places there will be in the terminating decimal number equivalent to each of these basic fractions.

a. $\frac{4}{25}$ b. $\frac{5}{8}$ c. $\frac{3}{50}$ d. $\frac{1}{40}$ e. $\frac{1}{200}$

Check your answers by turning to the Appendix.



Example 1

$\frac{5}{6}$ can mean $5 \div 6$.

$$\begin{array}{r} 0.8\ 3 \\ 6\overline{)5.0\ 0} \\ \underline{4\ 8} \\ 2\ 0 \\ \underline{1\ 8} \\ 2 \end{array}$$

← The remainder starts to repeat.

When dividing by 6, there are 5 possible remainders: 1, 2, 3, 4, and 5. This means that the maximum number of digits in the repeating block is 5. **Note:** Since only one remainder, 2, occurs, there is only one digit in the repeating block.

$\therefore \frac{5}{6} = 0.8\overline{3}$

Example 2

```
      0.2 8 5 7 1 4 2
   7)2.0 0 0 0 0 0 0
     1 4
       6 0
       5 6
         4 0
         3 5
           5 0
           4 9
             1 0
               7
               3 0
               2 8
                 2 0
                 1 4
                   6  <-- The remainder starts to repeat.
```

$\frac{2}{7}$ can mean $2 \div 7$.

There are six different remainders.

When dividing by 7, there are 6 possible remainders: 1, 2, 3, 4, 5, and 6. This means that the maximum number of digits in the repeating block is 6.

Note: Since all 6 remainders occurred, there are 6 digits in the repeating block.

$\therefore \frac{2}{7} = 0.\overline{285\,714}$

Example 3

What is the maximum number of digits in the repeating block of the repeating decimal number equal to $\frac{5}{12}$?

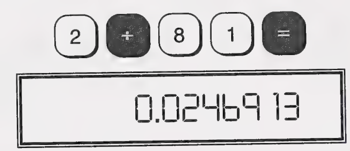
Solution

When dividing by 12, there are 11 possible remainders. This means that the maximum number of digits in the repeating block is 11.

6. If each of these fractions is expressed as a repeating decimal number, what is the maximum number of digits in the repeating block?

a. $\frac{6}{11}$ **b.** $\frac{2}{13}$ **c.** $\frac{5}{18}$ **d.** $\frac{4}{15}$

7. To change $\frac{2}{81}$ to a decimal fraction, Freddy used a calculator.



Freddy realized that $\frac{2}{81}$ can be expressed as a repeating decimal number because $\frac{2}{81} = \frac{2}{3 \times 3 \times 3 \times 3}$. He concluded that $\frac{2}{81} = 0.\overline{024\,691\,3}$.

Was Freddy's reasoning correct? Why or why not?

Check your answers by turning to the Appendix.

Conclusion

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This section helped to develop your sense for fractions and decimal numbers, the different ways fractions and decimal numbers can be expressed, and the sizes of various fractions and decimal numbers.

You found equivalent forms for fractions and decimal numbers. You expressed numbers less than one as proper fractions and decimal numbers. You expressed numbers greater than one as improper fractions, mixed numbers, and decimal numbers. You also ordered fractions and decimal numbers.

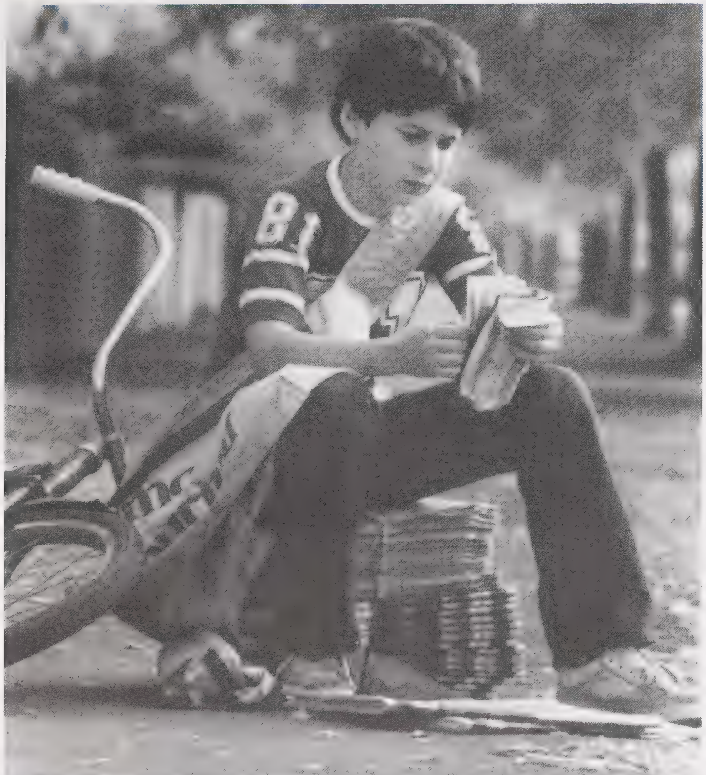
You were shown many situations where fractions and decimal numbers are used. Were you surprised to discover that fractions are used in music?

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 1.

Section 2: Operations with Decimal Numbers



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Many junior high students earn money by delivering newspapers. Others babysit, mow lawns, or help with recycling. Do you have a part-time job?

In order to work with money, you must be able to perform operations on decimal numbers.

In this section you will use concrete materials to get a sense of operations and what it means to add, subtract, multiply, and divide decimal numbers.

You will estimate and compute exact answers. You will add, subtract, multiply, and divide decimal numbers in several ways—using paper and pencil, a calculator, and mental math.

Activity 1: Adding and Subtracting

Slalom skiers twist and turn their way through a series of gates while skiing down a mountainside. The goal is to ski through the gates in the shortest possible time.

In the Olympics, slalom skiers race in two runs. The first run is in the morning, and the second run is four to five hours later. The gold medal winner is the racer who has the lowest combined time for the two heats. The clocks which measure the times for the slalom heats are very accurate. Since 1964, times have been measured to the nearest hundredth of a second. So, the total time is found by adding the two decimal numbers.

Adding decimal numbers is similar to adding whole numbers; you must line up the digits in each decimal place.

Examples 1 and 2 show the relationship between the concrete models and the paper-and-pencil method. Base ten blocks are used to model the sums. (A flat represents 1, a long represents 0.1, and a small cube represents 0.01.)

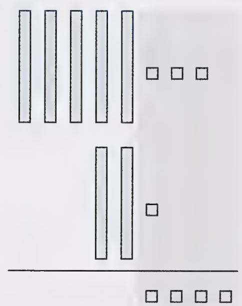
Example 1

Evaluate $0.53 + 0.21$.

Solution

Step 1: Add the hundredths.

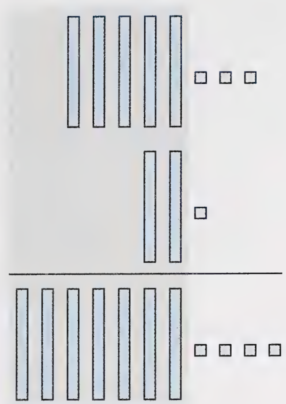
Concrete Model



Paper-and-Pencil Method

$$\begin{array}{r} 0.5\ 3 \\ +\ 0.2\ 1 \\ \hline 4 \end{array}$$

Step 2: Add the tenths.

Concrete Model	Paper-and-Pencil Method
	$\begin{array}{r} 0.\ 5\ 3 \\ +\ 0.\ 2\ 1 \\ \hline .\ 7\ 4 \end{array}$

Step 3: Add the ones.

Concrete Model	Paper-and-Pencil Method
There are no ones.	$\begin{array}{r} 0\ .53 \\ +\ 0\ .21 \\ \hline 0\ .74 \end{array}$

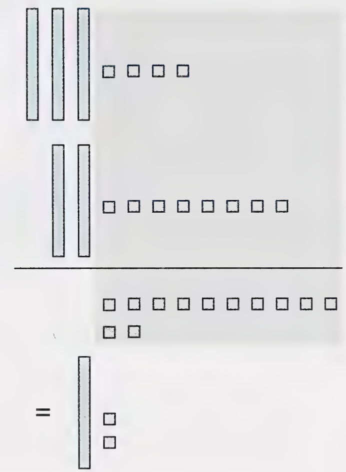
$\therefore\ 0.53 + 0.21 = 0.74$

Example 2

Evaluate $0.34 + 0.28$.

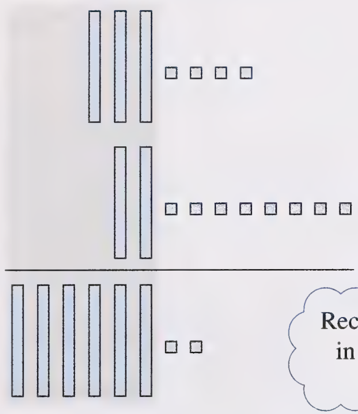
Solution

Step 1: Add the hundredths.

Concrete Model	Paper-and-Pencil Method
	$\begin{array}{r} 0.\overset{1}{3}\ 4 \\ +\ 0.2\ 8 \\ \hline 2 \end{array}$

Regrouping is required.

Step 2: Add the tenths.

Concrete Model	Paper-and-Pencil Method
	$\begin{array}{r} \overset{1}{0.\,3}\,4 \\ +\ 0.\,2\,8 \\ \hline .\,6\,2 \end{array}$

Recall that the regrouping in Step 1 resulted in an additional long.

Step 3: Add the ones.

Concrete Model	Paper-and-Pencil Method
There are no ones.	$\begin{array}{r} \overset{1}{0.\,3}4 \\ +\ 0.\,28 \\ \hline 0.\,62 \end{array}$

$\therefore\ 0.34 + 0.28 = 0.62$

The concrete models for Examples 1 and 2 helped me understand the paper-and-pencil method.

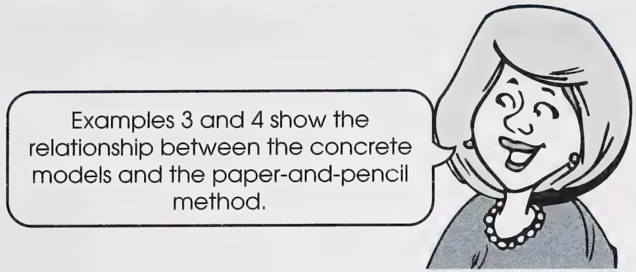
1. Evaluate each of the following sums using a concrete model and the paper-and-pencil method. Then compare the paper-and-pencil method with the model.

a. $0.12 + 0.35$ **b.** $0.54 + 0.67$

Check your answers by turning to the Appendix.

You are now ready to subtract decimal numbers.

Subtracting decimal numbers is very similar to subtracting whole numbers; you must line up the digits in each decimal place.



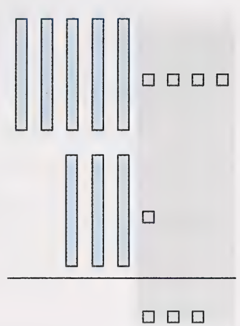
Example 3

Evaluate $0.54 - 0.31$.

Solution

Step 1: Subtract the hundredths.

Concrete Model

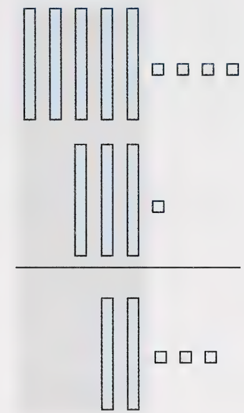


Paper-and-Pencil Method

$$\begin{array}{r} 0.5\ 4 \\ -\ 0.3\ 1 \\ \hline 3 \end{array}$$

Step 2: Subtract the tenths.

Concrete Model



Paper-and-Pencil Method

$$\begin{array}{r} 0.\ 5\ 4 \\ -\ 0.\ 3\ 1 \\ \hline .\ 2\ 3 \end{array}$$

Step 3: Subtract the ones.

Concrete Model

There are no ones.

Paper-and-Pencil Method

$$\begin{array}{r} 0\ .54 \\ -\ 0\ .31 \\ \hline 0\ .23 \end{array}$$

$\therefore\ 0.54 - 0.31 = 0.23$

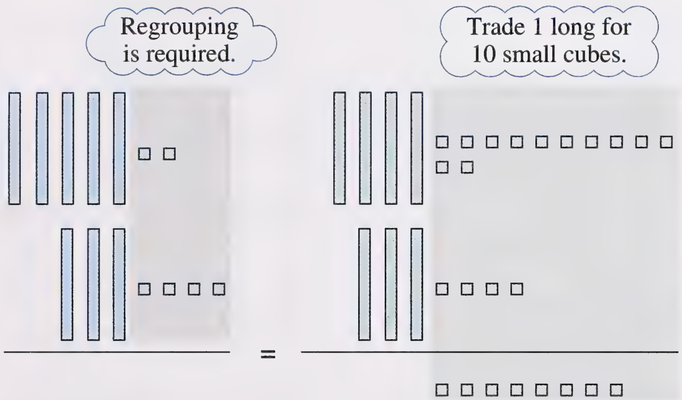
Example 4

Evaluate $0.52 - 0.34$.

Solution

Step 1: Subtract the hundredths.

Concrete Model



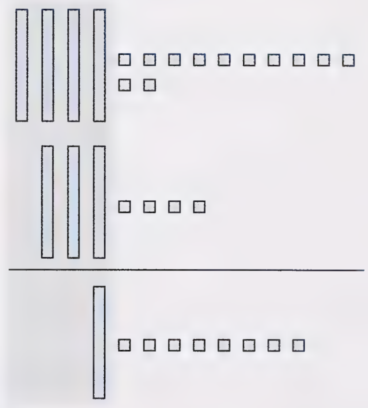
Paper-and-Pencil Method

$$\begin{array}{r} \scriptstyle 4\ 12 \\ 0.\cancel{5}\,\cancel{2} \\ -\ 0.3\ 4 \\ \hline 8 \end{array}$$

← Regroup numbers.

Step 2: Subtract the tenths.

Concrete Model



Paper-and-Pencil Method

$$\begin{array}{r} \scriptstyle 4\ 12 \\ 0.\ \cancel{5}\ \cancel{2} \\ -\ 0.\ 3\ 4 \\ \hline .\ 1\ 8 \end{array}$$

Step 3: Subtract the ones.

There are no ones.

$\therefore\ 0.52 - 0.34 = 0.18$

2. Evaluate each of the following differences using a concrete model and the paper-and-pencil method. Then compare the paper-and-pencil method with the concrete model.

a. $0.87 - 0.65$ **b.** $0.92 - 0.78$

Check your answers by turning to the Appendix.

Estimating

When you encounter problems involving the addition and subtraction of decimal numbers, you should always begin by estimating the answer. There are many estimation methods. The most commonly used methods for addition and subtraction are **front-end digits** and **rounding**.

Front-end digits is an estimating method in which you use only the first digit or digits in each number and zeros for place holders. Rounding is an estimating method in which you express each number to one of the following places: nearest whole number, nearest tenth, nearest hundredth,… .

Example 1

The four sections of the Wood Mountain Hiking Trail measure 1.92 km, 2.83 km, 3.14 km, and 2.71 km. What is the total length of the hiking trail?

Solution

Step 1: Estimate the sum by rounding to the nearest whole kilometre or using front-end digits.

Rounding

$$\begin{array}{r} 1.92 \\ 2.83 \\ 3.14 \\ +\ 2.71 \\ \hline \end{array} \doteq \begin{array}{r} 2 \\ 3 \\ 3 \\ +\ 3 \\ \hline 11 \end{array}$$

$\doteq$ means "approximately equal to."

Front-end Digits

$$\begin{array}{r} 1\ .92 \\ 2\ .83 \\ 3\ .14 \\ +\ 2\ .71 \\ \hline \end{array} \doteq \begin{array}{r} 1 \\ 2 \\ 3 \\ +\ 2 \\ \hline 8 \end{array}$$

Step 2: Calculate the sum.

$$\begin{array}{r} {\scriptstyle 2\ 1}\ \ \\ 1.92 \\ 2.83 \\ 3.14 \\ +\ 2.71 \\ \hline 10.60 \end{array} = 10.6$$

Regrouping may be done mentally.

Step 3: Compare the calculated answer to the estimate.

Rounding

$10.6 \doteq 11$

Front-end Digits

$10.6 \doteq 8$

The answer is reasonable.

The trail is 10.6 km long.

Example 2

Jennifer's purchases at the bookstore totalled \$8.21. She gave the salesclerk a \$10.00 bill. How much change did Jennifer receive?

Solution

Step 1: Estimate the difference by rounding to the nearest whole dollar or using front-end-digits.

Rounding

$$\begin{array}{r} 10.00 \\ -\ 8.21 \\ \hline \end{array} \doteq \begin{array}{r} 10 \\ -\ 8 \\ \hline 2 \end{array}$$

Front-end Digits

$$\begin{array}{r} 10\ .00 \\ -\ 8\ .21 \\ \hline \end{array} \doteq \begin{array}{r} 10 \\ -\ 8 \\ \hline 2 \end{array}$$

Step 2: Calculate the difference.

$$\begin{array}{r} {}^{9}\ \ {}^{9}\ \ \\ \cancel{10}\ \cancel{10}10 \\ \cancel{1}\cancel{0}.\cancel{0}0 \\ -\ 8.2\,1 \\ \hline 1.7\,9 \end{array}$$

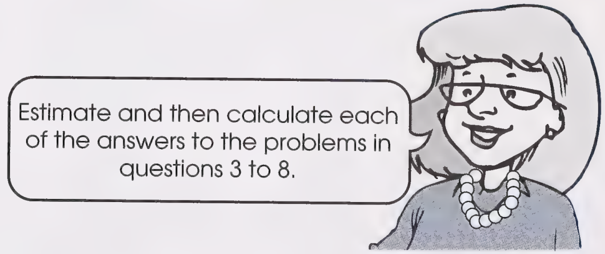
← Regrouping may be done mentally.

Step 3: Compare the calculated answers to the estimate.

$1.79 \doteq 2$

The answer is reasonable.

Jennifer received \$1.79 in change.

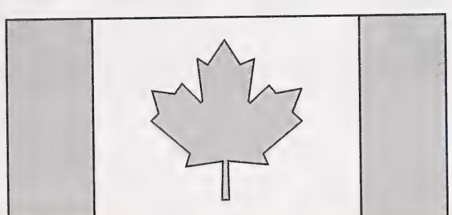


3. A developer bought three adjacent properties that had frontages of 56.2 m, 57.9 m, and 55.3 m. What was the total frontage of the three properties?

4. In the 1980 Olympics the gold medallist in the women's slalom was Hanni Wenzil from Liechtenstein. Her times were 42.50 s and 42.59 s. What was her total time?

5. In 1984, Canadian Victor Davis won an Olympic gold medal in the 200-m butterfly with a time of 133.34 s. In 1988, Hungarian Jozef Szabo won the gold in this event with a time of 133.52 s. How much faster was Davis' time than Szabo's?

6. In 1984, Canadian Sylvie Bernier won an Olympic gold medal in springboard diving by scoring 530.70. In 1988, Gao Min of China won the gold in this event by scoring 580.23. How much higher a score did the 1988 Olympic gold winner have than the 1984 Olympic gold winner?



7. Wendy bought juice for $1.25, a muffin for $0.85, and yogurt for $1.39. If the goods and service tax (GST) was included in these prices, what did Wendy pay altogether?

8. Roland purchased a pair of runners for $43.29, including tax. If he gave the clerk $50.00, how much change did he receive?

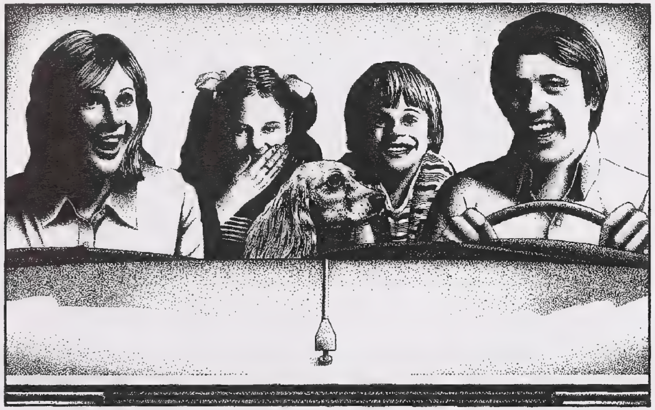
Check your answers by turning to the Appendix.

Using a Calculator

To avoid tedious calculations, you may use a calculator when solving problems. However, remember to estimate the answer first and use the estimate to check whether your answer is reasonable.

Example

The Ruhls and the Rajus bought cars at the same time. Five years later the odometer of the Ruhl's car read 98 765.8 km. The odometer on the Raju's car read 73 546.1 km. How much farther did the Ruhls drive?



Solution

Step 1: Estimate the answer by using rounding or front-end digits.

Rounding

$$\begin{array}{r} 98\,765.8 \\ -\ 73\,546.1 \\ \hline \end{array} \doteq \begin{array}{r} 100\,000 \\ -\ 70\,000 \\ \hline 30\,000 \end{array}$$

Front-end Digits

$$\begin{array}{r} 9\ 8\,765.8 \\ -\ 7\ 3\,546.1 \\ \hline \end{array} \doteq \begin{array}{r} 90\,000 \\ -\ 70\,000 \\ \hline 20\,000 \end{array}$$

Step 2: Calculate the answer.

9 8 7 6 5 • 8
− 7 3 5 4 6 • 1 =

252 19.7

Step 3: Compare the calculated answer to the estimate.

Rounding	Front-end Digits
$25\,219.7 \doteq 30\,000$	$25\,219.7 \doteq 20\,000$

The answer is reasonable.

The Ruhls drove 25 219.7 km farther.

Note: It is easy to make an error entering numbers on a calculator. Read your owner's manual to discover how the clearing function on your calculator works.

- On some calculators, pressing C erases the last entry and pressing AC clears the display.
- On other calculators, pressing CE erases the last entry and pressing C clears the display.

Use a calculator to answer questions 9 and 10. Be sure to estimate first.

9. Hugh bought a sweater for $28.20, a pair of jeans for $32.98, a pair of runners for $69.89, a pair of socks for $5.99, and a cap for $29.59. The tax was $11.67. What was Hugh's total bill?

10. The tallest free-standing structure in the world is the CN Tower in Toronto, Ontario. It is 553.34 m tall. The tallest guyed structure in the world was the Warszawa Radio mast in Poland. (It fell during renovation work in 1991.) It was 646.38 m tall.

How much taller was the Warszawa Radio mast than the CN Tower?

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Now Try This

Have you ever been to the top of the CN Tower in Toronto, Ontario? If not, use the Internet to discover what can be seen from the top of the CN Tower. This is the uniform resource locator (URL) for the CN Tower Picture Tour:

http.//www.epas.utoronto.ca:8080/epas/city_tour.html

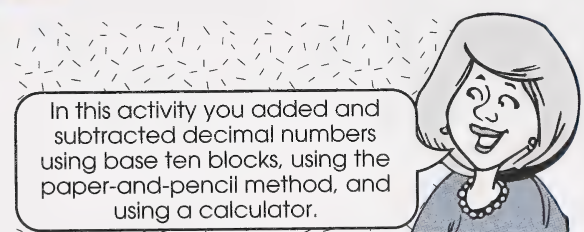
Use a problem-solving strategy to answer the following question.

11. In the following magic square, the sum of each row, column, and diagonal is 15.

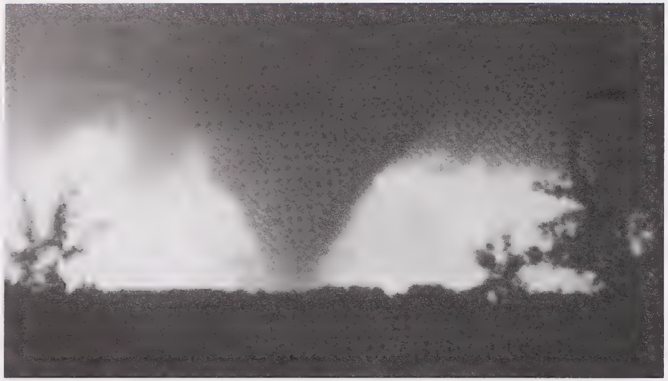
4	9	2
3	5	7
8	1	6

Change the magic square so that the sum of each row, column, and diagonal is 75.

Check your answers by turning to the Appendix.



Activity 2: Multiplying



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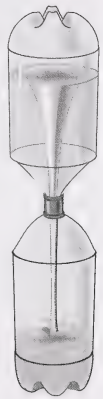


Have you studied tornadoes? If so, you may have used two plastic pop bottles and a special tube to visualize how the funnel-shaped tornadoes are formed.

Models are helpful in visualizing math concepts too.

In Activity 1 you used base ten blocks to model the addition and subtraction of decimal numbers.

In this activity you will use base ten blocks to model the multiplication of decimal numbers.

One Decimal Factor

Study the following examples to discover the meaning of multiplying a decimal number by a whole number.

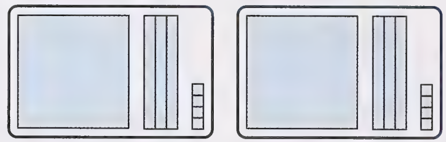
Example 1

Model and find the product of 2×1.34.

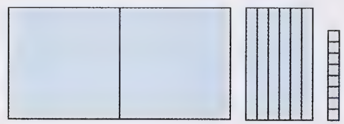
Solution

2×1.34 can mean two groups of 1.34.

Step 1: Model the two groups.



Step 2: Add the groups by combining the hundredths, the tenths, and the ones.



There are 2 flats,
6 longs, and
8 small cubes.

The product is 2.68.

Note: The product is found by repeated addition.

$$\begin{array}{r} 1.34 \\ \times \quad 2 \\ \hline \end{array} = \begin{array}{r} 1.34 \\ +\ 1.34 \\ \hline 2.68 \end{array}$$

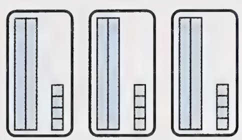
Example 2

Model and find the product of 3×0.24.

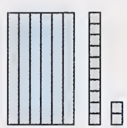
Solution

3×0.24 can mean three groups of 0.24.

Step 1: Model the three groups of 0.24.

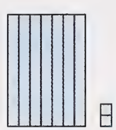


Step 2: Add the groups by combining the hundredths and the tenths.



There are 6 longs and 12 small cubes.

Step 3: Simplify the model by trading 10 small cubes for 1 long.



There are 7 longs and 2 small cubes.

The product is 0.72.

Note: The product is found by repeated addition.

$$\begin{array}{r} 0.24 \\ \times \quad 3 \\ \hline \end{array} = \begin{array}{r} \overset{1}{0.24} \\ 0.24 \\ + \ 0.24 \\ \hline 0.72 \end{array}$$

← Regrouping may be done mentally.

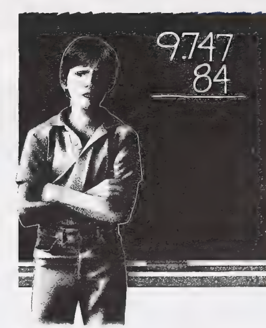
1. Use base ten blocks or the cut-out learning aids to model and find the product of each of the following problems.

a. 3×1.12 **b.** 2×1.35

Check your answers by turning to the Appendix.

Now that you established the meaning of multiplying a decimal number by a whole number, you are ready for the paper-and-pencil method.

Multiplying a decimal number by a whole number is similar to multiplying whole numbers. However, with decimal numbers the decimal points are not lined up as they are in addition and subtraction problems. So, you will need to know where to place the decimal point in the product.



The number of decimal places in the product is equal to the sum of the decimal places in the factors.

Example 3

At the fair the price of a hot dog is \$1.79. If Jeremy bought two hot dogs, how much did he pay?

Solution

Step 1: Estimate the answer by rounding or using front-end digits.

Rounding

$$\begin{array}{r} 1.79 \\ \times \quad 2 \\ \hline \end{array} \doteq \begin{array}{r} 2 \\ \times\ 2 \\ \hline 4 \end{array}$$

Front-end Digits

$$\begin{array}{r} 1\ .79 \\ \times \quad 2 \\ \hline \end{array} \doteq \begin{array}{r} 1 \\ \times\ 2 \\ \hline 2 \end{array}$$

Step 2: To calculate the exact answer, multiply as you would with whole numbers.

$$\begin{array}{r} 1.79 \\ \times \quad 2 \\ \hline 358 \end{array}$$

Step 3: Place the decimal point in the product.

$$\begin{array}{rl} 1.\ 79 & \leftarrow \text{2 decimal places} \\ \times \quad 2 & \leftarrow \text{0 decimal places} \\ \hline 3.\ 58 & \leftarrow \text{2 decimal places} \end{array}$$

$$\begin{array}{rl} & \text{2 decimal places} \\ + & \text{0 decimal places} \\ \hline & \text{2 decimal places} \end{array}$$

Step 4: Compare the calculated answer to the estimate.

Rounding

$3.58 \doteq 4$

Front-end Digits

$3.58 \doteq 2$

The answer is reasonable.

Jeremy paid \$3.58.

Example 4

A bus load of 24 tourists paid \$5.25 each for a tour of the city. How much did they pay altogether?

Solution

Step 1: Estimate the answer by rounding or by using front-end digits.

Rounding

$$\begin{array}{r} 5.25 \\ \times \quad 24 \\ \hline \end{array} \doteq \begin{array}{r} 5 \\ \times\ 24 \\ \hline 120 \end{array}$$

$24 \times 5 = 5 \times 24$

Front-end Digits

$$\begin{array}{r} 5\ .2\ 5 \\ \times \quad 2\ 4 \\ \hline \end{array} \doteq \begin{array}{r} 5 \\ \times\ 20 \\ \hline 100 \end{array}$$

Use zero as a place holder.

Step 2: To calculate the exact answer, multiply as you would with whole numbers.

$$\begin{array}{r} 5.25 \\ \times \quad 24 \\ \hline 2100 \\ 1050 \\ \hline 12600 \end{array}$$

Step 3: Place the decimal point in the product.

$$\begin{array}{rl} 5.25 & \leftarrow \text{2 decimal places} \\ \times \quad 24 & \leftarrow \text{0 decimal places} \\ \hline 2100 & \\ 1050 & \\ \hline 126.00 & \leftarrow \text{2 decimal places} \end{array}$$

$$\begin{array}{rl} & \text{2 decimal places} \\ + & \text{0 decimal places} \\ \hline & \text{2 decimal places} \end{array}$$

Step 4: Compare the calculated answer to the estimate.

Rounding	Front-end Digits
$120 \doteq 126$	$100 \doteq 126$

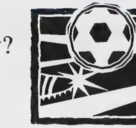
The answer is reasonable.

The tourists paid \$126.

2. Use estimation and the paper-and-pencil method to solve the following problems.

a. A stamp costs \$0.45. How much do 12 stamps cost?

b. A soccer ball costs \$12.98. If a soccer team bought 4 soccer balls, how much did they pay?



Check your answers by turning to the Appendix.

Two Decimal Factors

The product of two factors can be shown as a **rectangular array**.

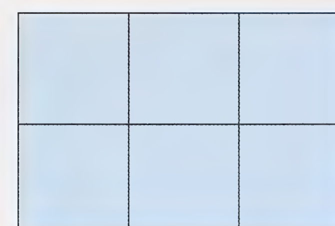
A rectangular array is an arrangement in columns and rows.

Example 1

Model and find the product of 2×3.

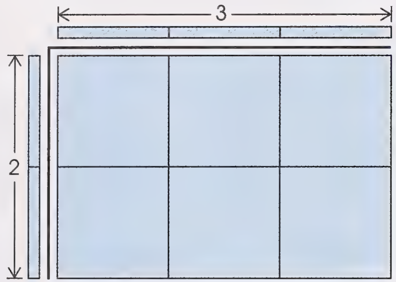
Solution

You can show 2×3 as a rectangular array with factors of 2 and 3.



The product is 6.

Note: To emphasize that the factors are 2 and 3, you can lay longs along the sides of the rectangle. (A long has the same length as a flat.)

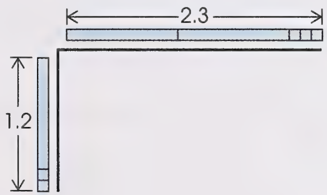


Example 2

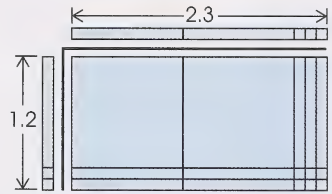
Model and find the product of 1.2×2.3.

Solution

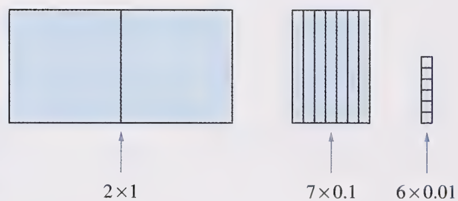
Step 1: Use longs and small cubes to show the factors.



Step 2: Build a rectangle using these factors as a guide. Use the fewest base ten blocks possible.



Step 3: Find the product. It may be easier to do this if you rearrange the blocks in the rectangle and group the ones, tenths, and hundredths. (Put aside the blocks you used to show the factors.)



The product is 2.76.

Note: To find the product of 1.2 and 2.3, the hundredths, tenths, and ones of the rectangular array are added.

$$\begin{array}{r} 2.3 \\ \times\ 1.2 \\ \hline \end{array} = \begin{array}{r} 2. \\ .4 \\ .3 \\ +\ .06 \\ \hline 2.76 \end{array}$$

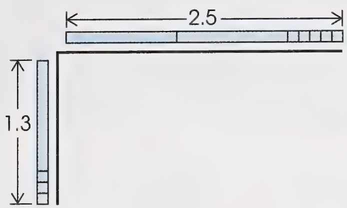
No regrouping is necessary.

Example 3

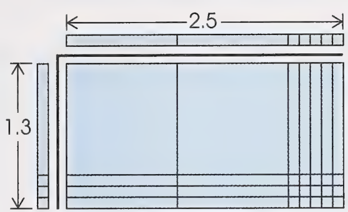
Model 1.3×2.5 as a rectangular array and then evaluate the product.

Solution

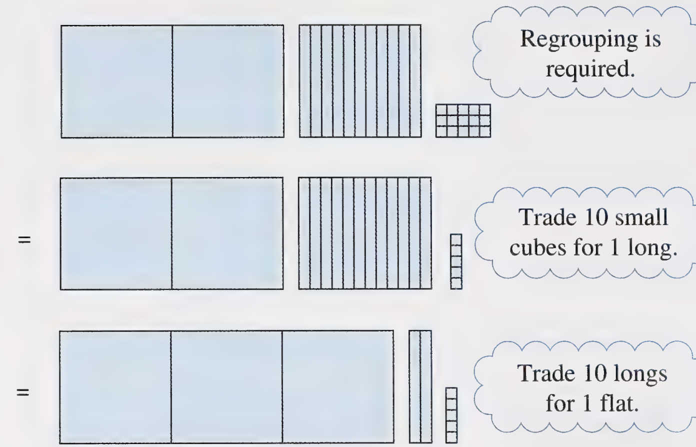
Step 1: Use longs and small cubes to show the factors.



Step 2: Build a rectangle using these factors as guides. Use the fewest blocks possible.



Step 3: Determine the product. You may find it easier to do this if you rearrange the blocks in the rectangular array. Group the ones, tenths, and hundredths. (Put aside the blocks you used to show the factors).



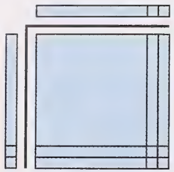
The product is 3.25.

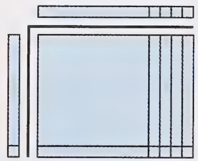
Note: To find the product, the hundredths, tenths, and ones of the rectangular array are added.

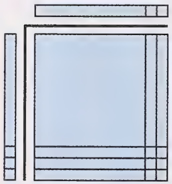
$$\begin{array}{r} 2.5 \\ \times\ 1.3 \\ \hline \end{array} = \begin{array}{r} 2. \\ .6 \\ .5 \\ +\ .15 \\ \hline 3.25 \end{array}$$

Regrouping is required.

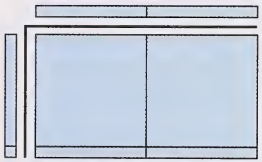
3. What are the factors in each of the following rectangular arrays?

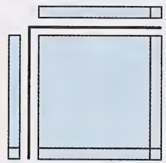
a.


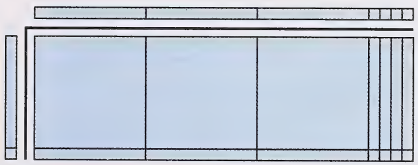
b.


c.


d.


e.


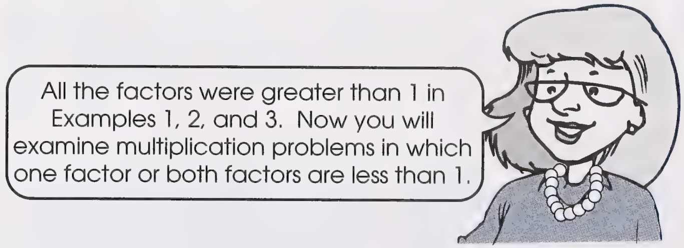
f.


g.


4. Use base ten blocks or the cut-out learning aids to model each of the following multiplication problems. Then find the products.

a. 1.2×1.1 b. 1.4×1.3 c. 1.5×1.2

Check your answers by turning to the Appendix.

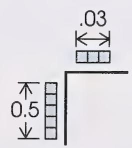


Example 4

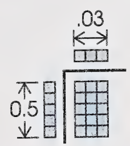
Model 0.5×0.3.

Solution

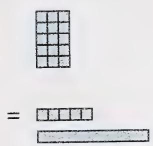
Step 1: Use small cubes to show the factors.



Step 2: Build a rectangle using these factors as guides. Use the fewest blocks possible.



Step 3: Determine the product. You may find it easier to do this if you rearrange the blocks in the rectangular array and group the tenths and hundredths. (Put aside the blocks you used to show the factors.)



Trade 10 small cubes for 1 long.

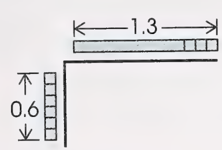
The product is 0.15.

Example 5

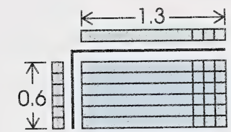
Model 0.6×1.3.

Solution

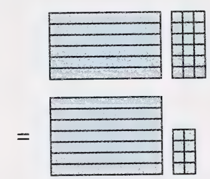
Step 1: Use longs and small cubes to show the factors.



Step 2: Build a rectangle using these factors as guides. Use the fewest blocks possible.



Step 3: Determine the product. You may find it easier to do this if you rearrange the blocks in the rectangular array and group the ones, tenths, and hundredths. (Put aside the blocks you used to show the factors.)



Trade 10 small cubes for 1 long.

The product is 0.78.

Note: To find the product, the hundredths, the tenths, and the ones of the rectangular array are added.

$$\begin{array}{r} 1.3 \\ \times\ 0.6 \\ \hline \end{array} = \begin{array}{r} .6 \\ +\ .18 \\ \hline 0.78 \end{array}$$

Regrouping is required.

5. Use base ten blocks or the cut-out learning aids to model each of the following multiplication problems. Then find the products.

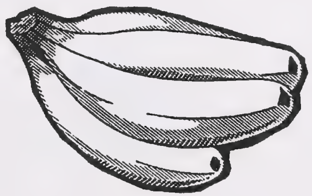
a. 0.5×0.4 **b.** 0.6×1.2 **c.** 0.3×1.4

Check your answers by turning to the Appendix.

Now that you have established the meaning of multiplication by two decimal factors, you are ready for the paper-and-pencil method.

Example 6

Louise bought 1.8 kg of bananas at the grocery store. If one kilogram of bananas costs \$1.07, how much did Louise pay for the bananas?



Solution

Step 1: Estimate the answer by rounding or by using front-end digits.

Rounding	Front-end Digits
$\begin{array}{r} 1.07 \\ \times\ 1.8 \\ \hline \end{array} \doteq \begin{array}{r} 1 \\ \times\ 2 \\ \hline 2 \end{array}$	$\begin{array}{r} 1\ .07 \\ \times\ 1\ .8 \\ \hline \end{array} \doteq \begin{array}{r} 1 \\ \times\ 1 \\ \hline 1 \end{array}$

Step 2: To calculate the exact answer, multiply as you would with whole numbers.

$$\begin{array}{r} 1.07 \\ \times\ 1.8 \\ \hline 856 \\ 107 \\ \hline 1926 \end{array}$$

Step 3: Place the decimal point in the product.

$$\begin{array}{rl} 1.\ 07 & \leftarrow \text{2 decimal places} \\ \times\ 1.\ 8 & \leftarrow \text{1 decimal place} \\ \hline 856 & \\ 107 & \\ \hline 1.\ 926 & \leftarrow \text{3 decimal places} \end{array}$$

$$\begin{array}{rl} & \text{2 decimal places} \\ + & \text{1 decimal place} \\ \hline & \text{3 decimal places} \end{array}$$

$$1.926 \doteq 1.93$$

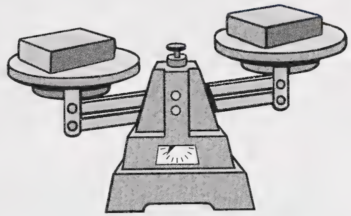
Step 4: Compare the calculated answer to the estimate.

Rounding	Front-end Digits
$1.93 \doteq 2$	$1.93 \doteq 1$

The answer is reasonable.

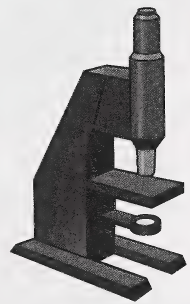
Louise paid \$1.93 for the bananas.

Estimate and then calculate the answers in questions 6 and 7.



6. A cubic centimetre of iron weighs 4.6 g. If gold is 2.4 times as heavy as iron, what is the mass of a cubic centimetre of gold?

7. With a microscope, scientists can compare the thickness of hair, wire, and red blood cells.



   a. A human hair is about 0.05 mm thick. If the wire to make coat hangers is about 7.5 times the thickness of a human hair, how thick is the wire of a coat hanger?

   b. A red blood cell is only 0.04 times the thickness of a human hair. How thick is a red blood cell?

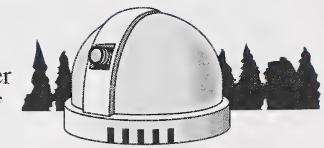
Check your answers by turning to the Appendix.

Using a Calculator

To avoid doing tedious calculations, you may use a calculator when solving problems. However, be sure to estimate the answer before using a calculator. Afterwards, compare the calculated answer to the estimate to decide if the answer is reasonable.

Example

By looking through powerful telescopes, scientists have determined that the diameter of Jupiter is 10.97 times the diameter of Earth. If Earth's diameter is 12 755 km, what is the diameter of Jupiter? Round the answer to the nearest kilometre.



Solution

Step 1: Estimate the answer using front-end digits.

$$\begin{array}{r} 1\ 2\,755 \\ \times \quad 1\ 0.97 \\ \hline \end{array} \doteq \begin{array}{r} 10\,000 \\ \times \quad 10 \\ \hline 100\,000 \end{array}$$

Zeros are used as place holders.

Step 2: To find the exact answer, use a calculator.

1 0 • 9 7 × 1 2 7 5 5 =

139922.35

Step 3: Compare the calculated answer to the estimate.

139 922.35 ≐ 100 000

The answer is reasonable.

The diameter of Jupiter is about 139 922 km.

Use a calculator to answer questions 8–10. (Estimate your answer before calculating.)

8. An elephant's skin is very thick. It is about 0.125 times the total mass of the elephant. If an elephant weighs 5.7 t, what is the mass of its skin?

The symbol "t" stands for tonnes.

9. A certain frog is 15.3 cm long. If the frog can jump 17.5 times the length of its body, how far can it jump?

10. The cruising speed of the supersonic jet, the Concorde, is 2.2 times the speed of sound. If the speed of sound is 1194 km per hour, what is the Concorde's cruising speed?

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

11. Copy the following problem and fill in the missing digits.

$$\begin{array}{rcccc} & \square & \square. & \square & \square \\ \times & & & \square. & 9 \\ \hline & 4 & \square & 0 & 1 \\ & \square & \square & \square & \square \\ \hline & \square & \square. & 4 & \square \end{array}$$

Check your answer by turning to the Appendix.

Did You Know?

François Viète (1540–1603), Simon Stevin (1548–1620), and John Napier (1550–1617) popularized the use of decimal fractions in Europe.

Use the Internet to find pictures of these mathematicians.

Word Search Puzzle

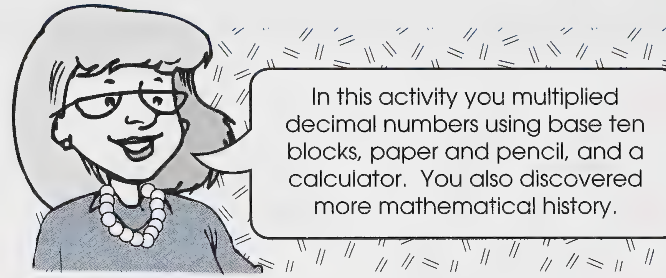
In the following word-search puzzle, find and circle the word FAMOUS and the word MATHEMATICIANS. Then find and circle the names in the given list. These names may be read forward, backward, diagonally, up, or down.

Great Names in Mathematics[1]

F	A	M	O	U	S	E	U	L	E	R
O	G	A	L	O	I	S	T	E	V	Y
S	O	T	E	R	E	N	O	P	S	C
E	T	H	B	M	G	O	D	E	L	H
N	H	E	R	O	N	T	D	L	O	R
E	A	M	V	S	O	E	R	L	G	I
H	L	A	L	I	M	L	E	B	A	E
T	E	T	R	I	N	E	E	A	U	M
S	S	I	H	N	E	C	T	B	S	A
O	H	C	P	E	A	N	O	G	S	N
T	R	I	O	W	N	O	T	W	E	N
A	O	A	L	T	A	P	I	O	L	E
R	T	N	K	B	R	I	G	G	S	A
E	N	S	T	R	E	U	C	L	I	D
N	A	P	I	E	R	R	V	E	N	N
E	C	A	U	C	H	Y	T	P	R	O

ABEL
ARCHIMEDES
BOOLE
BRIGGS
CANTOR
CAUCHY
ERATOSTHENES
EUCLID
EULER
GALOIS
GAUSS
GODEL
HERON
HILBERT
LIE
MONGE
NAPIER
NEWTON
PEANO
PELL
PONCELET
RIEMANN
STEVIN
THALES
VENN

[1] Reprinted with permission from *Mathematical History: Activities, Puzzles, Stories, and Games*. Copyright 1978 by the National Council of Teachers of Mathematics.



Activity 3: Dividing


[1]

To learn about collisions, crash-test dummies are used in remote-controlled cars. These models help show how injuries and speed are related. In this activity you will use base ten blocks to discover how multiplication and division are related.

These models will also give you a sense of what the operation of division means and help you estimate quotients.

[1] Transport Canada and the Ontario Ministry of Transportation and Communications. *The Human Collision*, 1976. Reproduced with the permission of the Minister of Supply and Services Canada, 1992.

Whole Number Divisors

Study the following examples to discover the meaning of dividing decimal numbers by a whole number. Notice the relationship between the concrete models and the paper-and-pencil method.

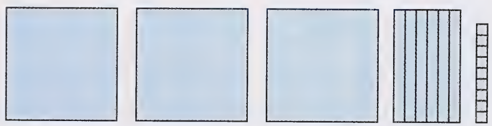
Example 1

What does $3.69 \div 3$ mean? Solve the problem.

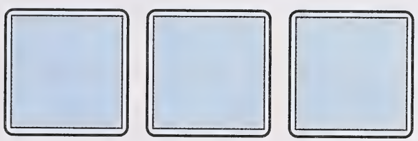
Solution

$3.69 \div 3$ can mean, "In 3.69 there are three groups of how many?"

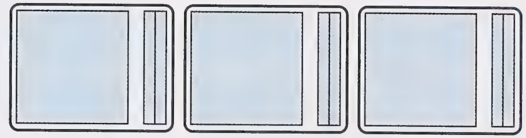
Step 1: Model the dividend of 3.69.



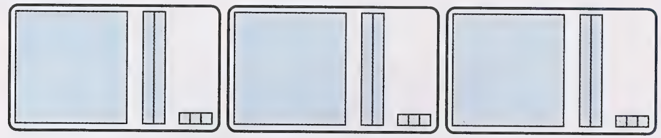
Step 2: Arrange the ones into three groups. There will be 6 tenths and 9 hundredths left over.



Step 3: Arrange the tenths into three groups. There will be 9 hundredths left over.



Step 4: Arrange the hundredths into three groups. There is no remainder.

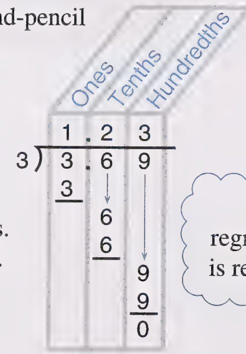


Each group has 1.23.

$\therefore\ 3.69 \div 3 = 1.23$

Note: The process for the paper-and-pencil method is similar to the process for modelling.

- Divide the 3 ones by 3.
- Bring down the 6 tenths.
- Divide the 6 tenths by 3.
- Bring Down the 9 hundredths.
- Divide the 9 hundredths by 3.



No regrouping is required.

Example 2

What does $2.36 \div 2$ mean? Solve the problem.

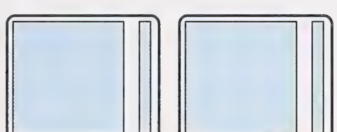
Solution

$2.36 \div 2$ can mean, "In 2.36 there are two groups of how many?"

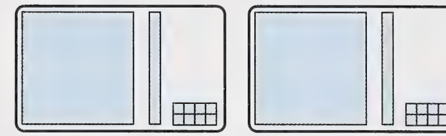
Step 1: Model the dividend of 2.36.

Step 2: Arrange the ones into two groups. There will be 3 tenths and 6 hundredths left over.

Step 3: Arrange the tenths into two groups. There will be 1 tenth and 6 hundredths left over.



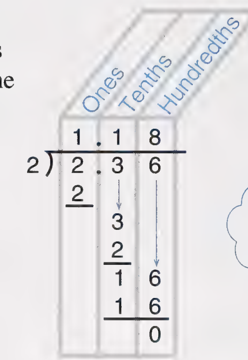
Step 4: Trade the leftover tenth for 10 hundredths so that there will be 16 hundredths altogether. Then arrange the hundredths into two groups.



There is 1.18 in each group.

$\therefore\ 2.36 \div 2 = 1.18$

Note: The process for the paper-and-pencil method is similar to the process for the model.



Regrouping is necessary.

- Divide the 2 ones by 2.
- Bring down the 3 tenths.
- Divide the 3 tenths by 2.
- Regroup the remainder of 1 tenth into 10 hundredths.
- Bring down the 6 hundredths and combine them with 10 hundredths to make 16 hundredths.
- Divide the 16 hundredths by 2.

1. Use base ten blocks or the cut-out learning aids from the Appendix to model each of the following division problems.

   a. $3.96 \div 3$ **b.** $2.58 \div 2$

Check your answers by turning to the Appendix.

Now that you have established the meaning of dividing a decimal number by a whole number, you are ready to use the paper-and-pencil method.

The first step in using the paper-and-pencil method is to estimate the answer. When you estimate a quotient, use **compatible numbers** that are close to the dividend and the divisor, and that divide exactly.

Compatible numbers are numbers that are easy to work with.

Example 3

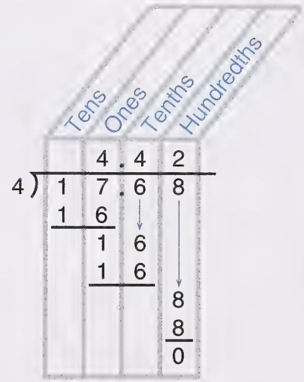
The cost of a pizza was shared equally by four friends. If the pizza cost $17.68, how much did each person pay?

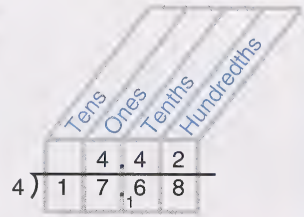
Solution

Step 1: Estimate the answer by using compatible numbers.

$$17.68 \div 4 \doteq 20 \div 4 \doteq 5 \quad \textbf{or} \quad 17.68 \div 4 \doteq 16 \div 4 \doteq 4$$

Step 2: Calculate the exact answer using long or short division.





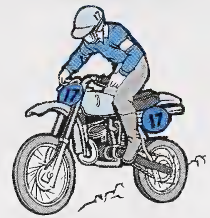
Step 3: Compare the calculated answer to the estimate.

$4.42 \doteq 5$ or $4.42 \doteq 4$

The answer is reasonable.

Each person paid $4.42.

Example 4



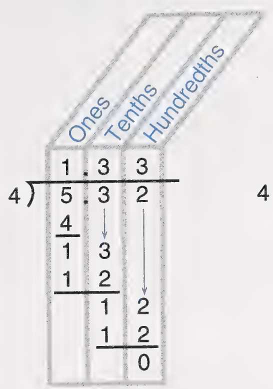
Francis won a race by riding his dirt bike around a track four times in 5.32 min. What was Francis' average time per lap?

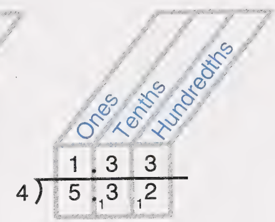
Solution

Step 1: Estimate the answer by using compatible numbers.

$$5.32 \div 4 \doteq 4 \div 4$$
$$\doteq 1$$

Step 2: Calculate the exact answer using long or short division.





Step 3: Compare the calculated answer to the estimate.

$$1.33 \doteq 1$$

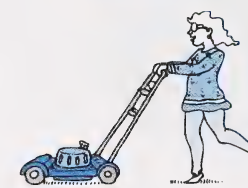
The answer is reasonable.

Francis' average time per lap was 1.33 min.

2. Use estimation and the paper-and-pencil method to solve each of the following problems.

a. A car uses 8 L of gasoline to travel 96.4 km. What is the average number of kilometres that the car can be driven on 1 L of gasoline?

b. A stack of 77 quarters is 123.2 mm high. How thick is a quarter?



c. In the summer Janice works 36.25 h each week mowing lawns. If she works 5 days each week, how many hours does she work each day?

Check your answers by turning to the Appendix.

Decimal Divisors

In the previous activity you used a rectangular array to model the product of two decimal factors. You can also use a rectangular array to show the meaning of a decimal divisor.

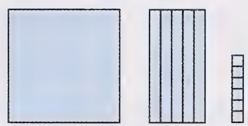
Example 1

Model $1.56 \div 1.3$ and find the quotient.

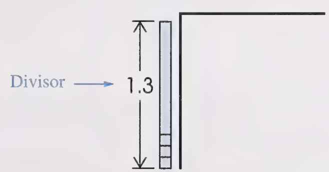
Solution

$1.56 \div 1.3 = \square$ is the same as $1.3 \times \square = 1.56$.

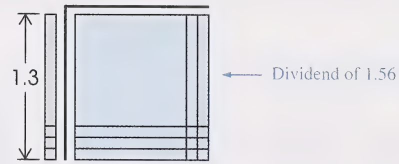
Step 1: Model the dividend of 1.56.



Step 2: Because the divisor is 1.3, there will be 1.3 rows in the rectangular array. Show this.

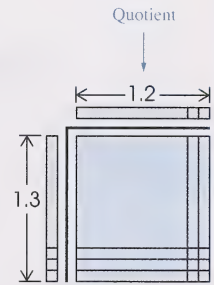


Step 3: Rearrange the model of the dividend so that a rectangular array with 1.3 rows is formed.



Step 4: To calculate the quotient, determine the number of columns in the rectangular array. Use 1 long and 2 small cubes to show this quotient.

There are 1.2 columns.



$\therefore\ 1.56 \div 1.3 = 1.2$

3. Use base ten blocks or the cut-out learning aids to model each of the following division problems.

a. $1.54 \div 1.1$ **b.** $1.68 \div 1.2$

Check your answers by turning to the Appendix.

The following example illustrates a division problem where the divisor is less than 1.

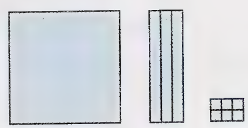
Example 2

Model $1.36 \div 0.4$ and find the quotient.

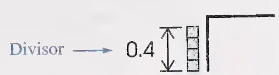
Solution

$1.36 \div 0.4 = \square$ is the same as $0.4 \times \square = 1.36$.

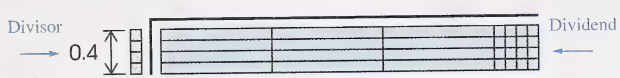
Step 1: Model the dividend of 1.36.



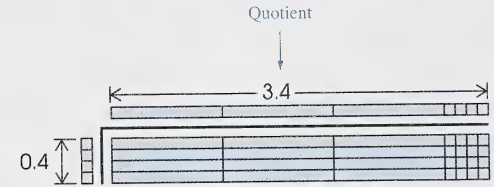
Step 2: Because the divisor is 0.4, there will be 0.4 rows in the rectangular array. Show this.



Step 3: Rearrange the model of the dividend so that a rectangular array with 0.4 rows is formed. To do this you must trade 1 flat for 10 longs, and trade 1 long for 10 small cubes.



Step 4: To calculate the quotient, determine the number of columns in the rectangular array. You may use 3 longs and 4 small cubes to show this.



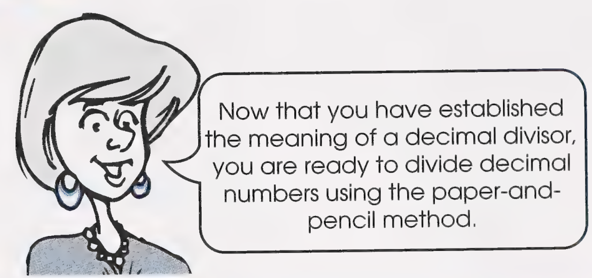
There are 3.4 columns.

$\therefore\ 1.36 \div 0.4 = 3.4$

4. Use base ten blocks or the cut-out learning aids to model each of the following division problems.

a. $1.92 \div 0.8$ **b.** $1.35 \div 0.5$

Check your answers by turning to the Appendix.



You should always estimate the answer before you calculate the exact answer. To estimate a quotient you can use compatible numbers.

Example 3

A stack of quarters is 44.8 mm high. If a quarter is 1.6 mm thick, how many quarters are in the stack?

Solution

Step 1: Estimate the answer by using compatible numbers.

$$\begin{aligned} 44.8 \div 1.6 &\doteq 44 \div 2 \\ &\doteq 22 \end{aligned}$$

Step 2: Multiply the dividend and the divisor by 10 to produce a whole number divisor. (Multiplying both the dividend and the divisor by the same number will not change the quotient.)

$$1.6\overline{)44.8} \longrightarrow \frac{44.8}{1.6} \overset{\times 10}{=} \frac{448}{16} \longrightarrow 16\overline{)448}$$

Step 3: Calculate the exact answer using long or short division.

	Hundreds	Tens	Ones
		2	8.
16)	4	4	8.
	3	2	↓
	1	2	8
	1	2	8
			0

	Hundreds	Tens	Ones
		2	8.
16)	4	4	^{12}8.

Step 4: Compare the calculated answer to the estimate.

$$28 \doteq 22$$

The answer is reasonable.

There are 28 quarters in the stack.

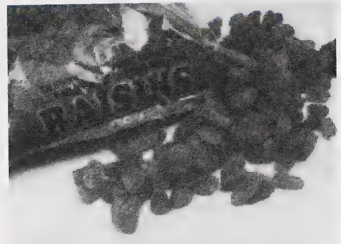
Note: You can use arrows or carets to show the new positions of the decimal points in the divisor and dividend.

$$\begin{array}{r} 28. \\ 1.6\overline{)44.8} \\ \underline{32} \\ 128 \\ \underline{128} \\ 0 \end{array} \qquad \begin{array}{r} 28. \\ 1.6_{\wedge}\overline{)44.8_{\wedge}} \\ \underline{32} \\ 128 \\ \underline{128} \\ 0 \end{array}$$

This is a caret.

Example 4

How many bags of raisins can be made from 25.5 kg of raisins if the raisins are packed in bags each containing 0.75 kg?



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Solution

Step 1: Estimate the answer by using compatible numbers.

$$\begin{aligned} 25.5 \div 0.75 &\doteq 26 \div 1 \\ &\doteq 26 \end{aligned}$$

Step 2: Multiply the dividend and the divisor by 100 to produce a whole number divisor. (Multiplying both the dividend and the divisor by the same number will not change the quotient.)

$$0.75\overline{)25.5} \longrightarrow \frac{25.5}{0.75} \underset{\times 100}{\overset{\times 100}{=}} \frac{2550}{75} \longrightarrow 75\overline{)2550}$$

Step 3: Calculate the exact answer using long or short division.

	Thousands	Hundreds	Tens	Ones
			3	4.
75)	2	5	5	0.
	2	2	5	↓
		3	0	0
		3	0	0
				0

	Thousands	Hundreds	Tens	Ones
			3	4.
75)	2	5	5	$_{30}$0.

Step 4: Compare the calculated answer to the estimate.

$34 \doteq 26$

The answer is reasonable.

A total of 34 bags can be made.

Note: You can use arrows or carets to show the new positions of the decimal points in the divisor and dividend.

$$\begin{array}{r} 3\ 4. \\ 0.75\overline{)25.5\ 0} \\ \underline{22\ 5} \\ 3\ 0\ 0 \\ \underline{3\ 0\ 0} \\ 0 \end{array}$$

The end zero is used as a place holder.

$$\begin{array}{r} 3\ 4. \\ 0.75_{\wedge}\overline{)25.5\ 0_{\wedge}} \\ \underline{22\ 5} \\ 3\ 0\ 0 \\ \underline{3\ 0\ 0} \\ 0 \end{array}$$

5. Estimate and then calculate the answers to the following problems.

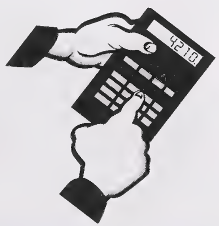
a. How many birdhouses can be built in 7.5 h if each birdhouse requires 1.5 h to build?

b. How many tetras (tropical fish) can you buy for \$9.00 if each tetra costs \$1.50?

c. The school day at Colchester Senior High is 6 h long. The day is divided into periods which last 0.75 h each. How many periods are in a school day?

Check your answers by turning to the Appendix.

Using a Calculator



To avoid doing tedious calculations, you may use a calculator when solving problems. However, be sure to estimate the answer before using a calculator.

Example

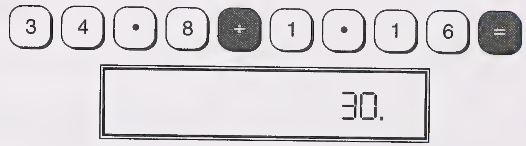
How many dimes are in a stack 34.8 mm high if a dime has a thickness of 1.16 mm?

Solution

Step 1: Estimate the answer by using compatible numbers.

$$\begin{aligned} 34.8 \div 1.16 &\doteq 35 \div 1 \\ &\doteq 35 \end{aligned}$$

Step 2: Compute the answer using a calculator.



Step 3: Compare the calculated answer to the estimate.

$30 \doteq 35$

The answer is reasonable.

The stack has 30 dimes in it.

Use a calculator to answer the problems in question 5.

6. Estimate and then calculate the answers to the following problems.

   a. A pile of quarters has a mass of 292.9 g. If one quarter has a mass of 5.05 g, how many quarters are in the pile?

   b. Marcia rode her dirt bike 31.25 km in 1.25 h. What was Marcia's average speed?

   c. Kim worked 36.25 h for $224.75. How much did he earn each hour?

Check your answers by turning to the Appendix.

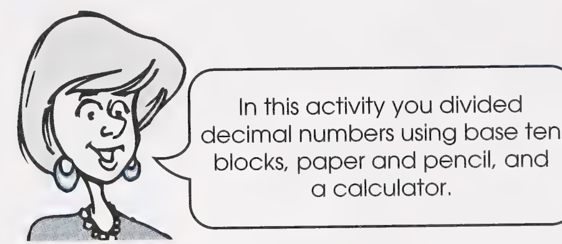
Now Try This

Use a problem-solving strategy to answer the following question.

7. Copy the given problem and fill in the missing digits.

```
        ▢.▢▢
25 ) ▢8.00
     ▢▢
     ▢8▢
     ▢▢▢
       ▢▢
       ▢▢
        0
```

Check your answer by turning to the Appendix.



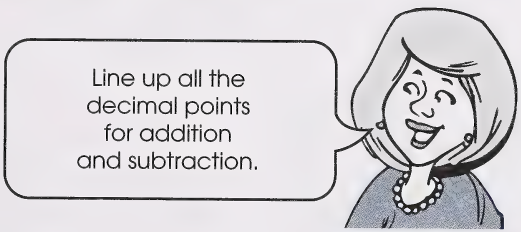
Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section you performed operations with decimal numbers. You may find it helpful to use graph paper to do the paper-and-pencil calculations. The graph paper will help you line up the digits correctly.

Adding and Subtracting



Example 1

Evaluate $4.8+0.35+1.2+3$.

Solution

$$\begin{array}{r} 4.80 \\ 0.35 \\ 1.20 \\ +\,3.00 \\ \hline 9.35 \end{array}$$

Line up the decimal points.

You may use zeros as place holders.

Example 2

Evaluate $9.7-2.43$.

Solution

$$\begin{array}{r} 9.70 \\ -\,2.43 \\ \hline 7.27 \end{array}$$

Line up the decimal points.

You may use zeros as place holders.

1. Find the following sums and differences.

a. $80.63+109.8$

b. $5.6+3.48+29.6+0.387$

c. $102.3-41.17$

d. $8.67-5.4$

Check your answers by turning to the Appendix.

Multiplying and Dividing

Example 1

Evaluate 3.82×1.4.

Solution

$$
\begin{array}{r}
3.82 \\
\times\quad 1.4 \\
\hline
1528 \\
382 \\
\hline
5.348
\end{array}
$$

Line up the digits in the factors flush right.

$$
\begin{array}{rl}
 & \text{2 decimal places} \\
+ & \text{1 decimal place} \\
\hline
 & \text{3 decimal places}
\end{array}
$$

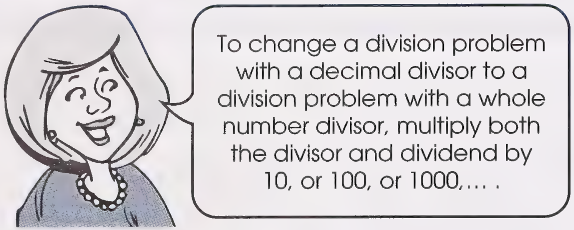
Example 2

Evaluate $14.58 \div 6$.

Solution

$$
\begin{array}{r}
2.43 \\
6\overline{)14.58} \\
\underline{12} \\
25 \\
\underline{24} \\
18 \\
\underline{18} \\
0
\end{array}
$$

Line up the decimal point in the quotient with the decimal in the dividend.



Example 3

Evaluate $0.406 \div 5.8$.

Solution

Step 1: Change this division problem to a problem with a whole number divisor.

$$5.8\overline{)0.406} \rightarrow \frac{0.406}{5.8} \underset{\times 10}{\overset{\times 10}{=}} \frac{4.06}{58} \rightarrow 58\overline{)4.06}$$

Step 2: Divide.

$$\begin{array}{r} 0.07 \\ 58\overline{)4.06} \\ \underline{4\,06} \\ 0 \end{array}$$

Line up the decimal point in the quotient with the decimal point in the dividend.

2. Find the following products.

   a. 0.68×7 **b.** 5.6×13 **c.** 3.7×0.6 **d.** 6.32×1.8

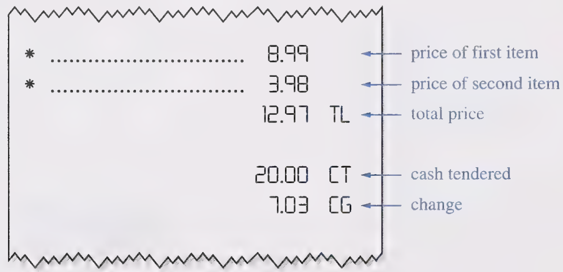
3. Find the following quotients.

   a. $6.6 \div 5$ **b.** $30.1 \div 43$
   c. $47.7 \div 0.09$ **d.** $36.9 \div 4.1$

Check your answers by turning to the Appendix.

Enrichment

This cash register receipt shows the price of each item purchased, the total (TL), the cash tendered (CT), and the change (CG).



In this activity you will work with cash register receipts. You will use different strategies to mentally compute the total price and the change.

Rounding and Adjusting

Studies show that more people will purchase an item that costs \$5.99 than one that costs \$6.00. That is why many prices end in 99 cents or 98 cents.

To mentally add prices ending in 99 cents or 98 cents, round up to the nearest dollar, add, and then adjust by subtracting the appropriate number of cents. This strategy is called **rounding and adjusting**.

Example 1

$$\begin{array}{r} 8.99 \\ +\ 3.98 \\ \hline \end{array}$$

To make change, you often subtract a decimal number from a whole number. To perform this operation mentally, you can use the rounding and adjusting strategy. Round the price up to the nearest dollar, subtract, and then adjust by adding the appropriate number of cents.

Example 2

$$\begin{array}{r} 20.00 \\ -\ 12.97 \\ \hline \end{array}$$

1. Use mental computation to find the total and the change for each of the following sales receipts.

a.

*	6.98	
*	2.97	
		TL
	20.00	CT
		CG

b.

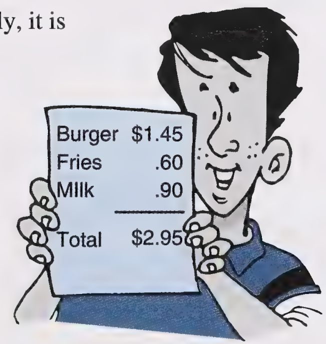
*	5.99	
*	6.99	
*	8.99	
		TL
	30.00	CT
		CG

Check your answers by turning to the Appendix.

Adding from the Left

When you add a list of prices using the paper-and-pencil method, you add from right to left. That is, you add the hundredths, the tenths, the ones,… .

When you add a list of prices mentally, it is often easier to add from the left.



This strategy is called the **left-to-right method** or **adding from the left**.

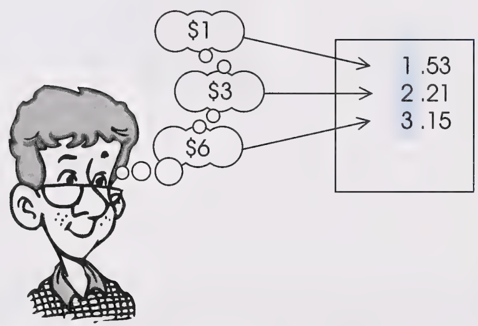
Example

Find the total cost of the three items on this cash register receipt.

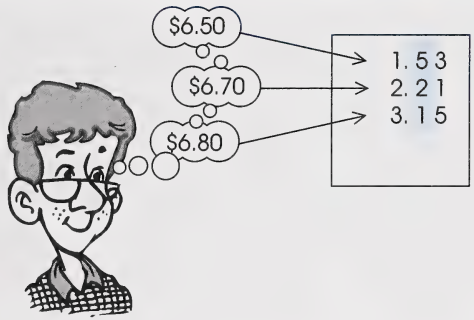
*	1.53	
*	2.21	
*	3.15	
		TL

Solution

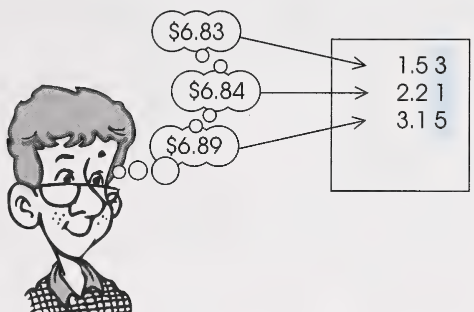
Step 1: Add the ones column.



Step 2: Add the tenths column.



Step 3: Add the hundredths column.



The total is $6.89.

2. Use mental computation to find the total of each of the following sales receipts.

a.

* 4.32
* 6.41
* 2.15

TL

b.

* 4.15
* 3.23
* 1.51

TL

c.

* 12.22
* 5.34
* 4.41

TL

Check your answers by turning to the Appendix.

Bridging

Another useful technique for adding prices mentally is **bridging**.

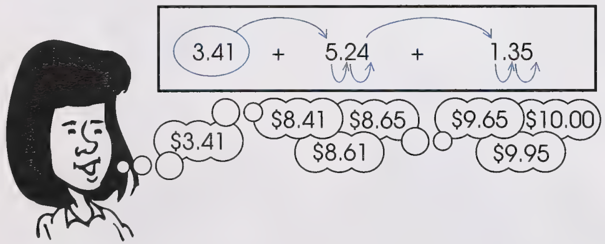
Bridging is a strategy in which you break up a number and add the parts, one at a time.

Example

Find the total cost of the three items on this cash register receipt.

* 3.41
* 5.24
* 1.35
TL

Solution



The total is $10.00.

3. Use mental computation to find the total of each list of prices.

a.

* 1.29
* 2.34
* 1.22
TL

b.

* 1.64
* 2.28
* 3.72
TL

c.

* 4.98
* 3.99
* 7.99
TL

Check your answers by turning to the Appendix.

Conclusion

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In this section you gained a sense of operations by modelling the addition, subtraction, multiplication, and division of decimal numbers. You performed exact calculations on paper and with a calculator. You checked the reasonableness of your answers by estimating. You also used mental math to add, subtract, multiply, and divide decimal numbers.

Chores like recycling bottles require skill in operations with decimals. For example, if you receive $0.20 for a 2-L bottle, how much do you get if you return 50 2-L bottles? If you want to earn $28.00, how many 2-L bottles will you need to collect?

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 2.

Module Summary

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This module helped you develop your sense of what a fraction is, the different ways fractions can be expressed, the relationship between fractions and decimal numbers, the sizes of various fractions and decimal numbers, and the meaning of operations on decimal numbers. You estimated and computed addition, subtraction, multiplication, and division problems. You computed with paper and pencil and a calculator, and you performed mental math.

This module also demonstrated some of the situations where fractions and decimal numbers are used. You discovered fractions and decimal numbers can be used in a wide variety of situations—from planning a pizza party, to composing music, to delivering newspapers, to recycling bottles.

Final Module Assignment

Assignment Booklet

You are now ready to complete the final module assignment for Module 2.

APPENDIX

- Glossary
- Suggested Answers
- Cut-out Learning Aids

Glossary

Adding from the left: a mental computation strategy in which you add columns of digits from left to right; also called the **left-to-right method**

Ascending order: in order from least to greatest

Basic fraction: a fraction in simplest form

Bridging: a mental computation strategy in which you break up numbers and add the parts, one at a time

Compatible numbers: an estimation strategy in which you use numbers that are easy to work with

Denominator: the bottom number of a fraction; the divisor

Descending order: in order from greatest to least

Ellipsis: a set of three dots indicating a continuing pattern

Equivalent decimals: decimals that name the same part of a whole

Equivalent fractions: fractions that name the same part of a whole

Expanded form: (of a decimal number) the form of the number expressed as a sum of products

Each product shows a digit times its place value, e.g.,

$$2.47 = 2 \times 1 + 4 \times 0.1 + 7 \times 0.01$$

Front-end digits: an estimating method in which only the first digit or digits in each number is used and zeros are place holders

Improper fractions: fractions in which the numerator is greater than the denominator

Left-to-Right method: a mental computation strategy in which you add columns of digits from left to right; also called **adding from the left**

Mixed numbers: numbers expressed as a sum of a whole number and a proper fraction

Numerator: the top number of a fraction; the dividend

Order: list numbers in order of size

Problem: a task for which the method of finding the answer (as well as the answer) is not immediately known

Proper fractions: fractions in which the numerator is less than the denominator

Rectangular array: an arrangement in columns and rows

Repeating decimal number: a decimal number with an infinite number of digits and a repeating pattern

Rounding: an estimating method in which a number is expressed to the nearest whole, nearest tenth, …

Rounding and adjusting: a mental computation strategy in which you round decimal numbers to the nearest whole number and adjust

Simplest form: (of a fraction) a fraction with the smallest possible whole-number denominator; also called a **basic fraction**

Standard form: (of a decimal number) the usual form of the number, for example, 2.47

Terminating decimal: a decimal with a finite number of digits

Vinculum: the bar in a fraction; also, the bar over the repeating block of digits in a repeating decimal number

Suggested Answers

Section 1: Activity 1

1. a. $\frac{7}{12}$ b. $\frac{19}{24}$

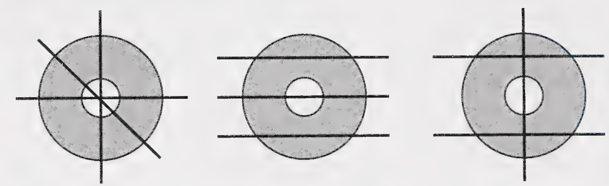
2. a. $\frac{3}{11}$ b. $\frac{1}{11}$

3. The number of full ice-cream cartons is $\frac{13}{6}$ or $2\frac{1}{6}$.

4. The fraction of cartons containing doughnuts is $\frac{17}{12}$ or $1\frac{5}{12}$.

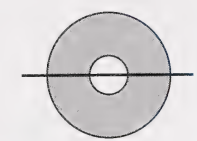
Fraction Challenge

5. Changing your point of view may help you solve this problem. You probably tried traditional ways of cutting the doughnuts, like this:

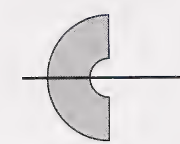


To solve the problem you have to shift your "mind set" and try a different approach. Here is one way to solve the problem.

Step 1: Cut the doughnut into two halves with one cut.

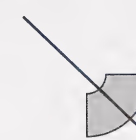


Step 2: Stack the two halves one on top of the other, and cut them into fourths with one cut.



There are now four-fourths.

Step 3: Stack the fourths on top of each other and cut them into eighths with one cut.



There are now eight-eighths.

Were you able to find another way?

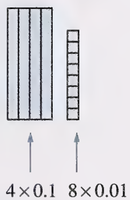
Section 1: Activity 2

1. **a.** 0.4; 4 tenths **b.** 2.5; 2 **and** 5 tenths
   c. 0.43; 43 hundredths **d.** 1.58; 1 **and** 58 hundredths
   e. 0.06; 6 hundredths **f.** 1.30; 1 **and** 30 hundredths

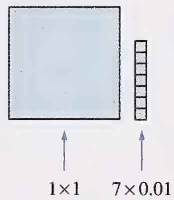
2. **a.** $1.25 = 1\times1 + 2\times0.1 + 5\times0.01$
   b. $2.07 = 2\times1 + 0\times0.1 + 7\times0.01$
   c. $3.40 = 3\times1 + 4\times0.1 + 0\times0.01$
   d. $0.57 = 0\times1 + 5\times0.1 + 7\times0.01$

3. **a.**

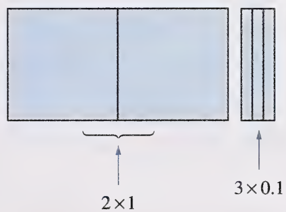
b.



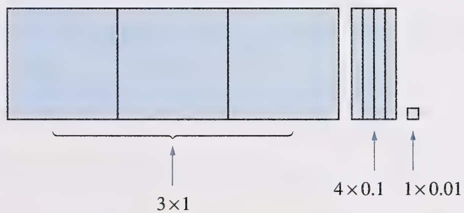
c.



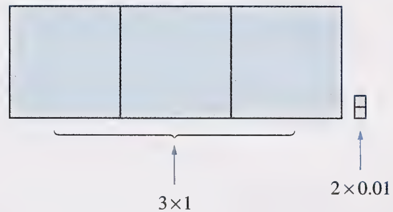
d.



e.



f.



4. **a.** $31.035 = 3\times10 + 1\times1 + 0\times0.1 + 3\times0.01 + 5\times0.001$; 31 and 35 thousandths

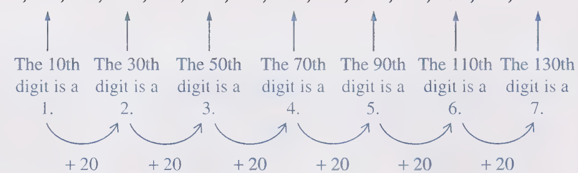
   b. $12.0005 = 1\times10 + 2\times1 + 0\times0.1 + 0\times0.01 + 0\times0.001 + 5\times0.0001$; 12 and 5 ten thousandths

   c. $1.000\,36 = 1\times1 + 0\times0.1 + 0\times0.01 + 0\times0.001 + 3\times0.0001 + 6\times0.000\,01$; 1 and 36 hundred thousandths

Now Try This

5. To solve this problem you may use patterns.

1, 2, 3, …, 10, …, 20, …, 30, …, 40, …, 50, …, 60, …, 70



The 130th digit is a **7**.

Section 1: Activity 3

1. a. $1=\frac{10}{10}$ b. $\frac{2}{3}=\frac{4}{6}$

2. $\frac{5}{12}=\frac{10}{24}=\frac{15}{36}=\frac{20}{48}=\frac{25}{60}$

3. a. $\frac{9}{12}=\frac{3}{4}$ b. $\frac{4}{6}=\frac{2}{3}$ c. $\frac{24}{96}=\frac{1}{4}$

   d. $\frac{72}{30}=\frac{12}{5}$ e. $\frac{30}{24}=\frac{5}{4}$ f. $\frac{45}{40}=\frac{9}{8}$

4. a. $5\frac{1}{4}=\frac{21}{4}$ b. $3\frac{2}{5}=\frac{17}{5}$ c. $2\frac{1}{3}=\frac{7}{3}$ d. $4\frac{3}{5}=\frac{23}{5}$

5. a. $\frac{10}{3}=3\frac{1}{3}$ b. $\frac{16}{5}=3\frac{1}{5}$ c. $\frac{21}{4}=5\frac{1}{4}$ d. $\frac{33}{2}=16\frac{1}{2}$

6. Answers may vary. Any number of zeros may be added to the end of the number.

   a. $0.60=0.6=0.600$ b. $0.7=0.70=0.700$
   c. $1.280=1.28=1.2800$ d. $19.05=19.050=19.0500$

7. a. $0.3=\frac{3}{10}$ b. $0.26=\frac{26}{100}=\frac{13}{50}$ c. $0.05=\frac{5}{100}=\frac{1}{20}$

   d. $4.25=4\frac{25}{100}=4\frac{1}{4}$ e. $2.875=2\frac{875}{1000}=2\frac{7}{8}$ f. $3.036=3\frac{36}{1000}=3\frac{9}{250}$

8. a. $\frac{3}{4}=\frac{75}{100}=0.75$ b. $\frac{2}{5}=\frac{4}{10}=0.4$ c. $\frac{5}{8}=\frac{625}{1000}=0.625$

   d. $1\frac{3}{10}=1.3$ e. $2\frac{1}{2}=2\frac{5}{10}=2.5$

9. a. $\frac{7}{9}=0.\overline{7}$ b. $\frac{5}{12}=0.41\overline{6}$ c. $\frac{9}{16}=0.5625$

   d. $\frac{9}{11}=0.\overline{81}$ e. $\frac{3}{40}=0.075$ f. $\frac{2}{13}=0.\overline{153846}$

Note: The numbers in c. and e. are terminating decimals.

10. a. $\frac{1}{9}=0.\overline{1}$, $\frac{2}{9}=0.\overline{2}$, $\frac{3}{9}=0.\overline{3}$, $\frac{4}{9}=0.\overline{4}$

    b. $0.\overline{1}$, $0.\overline{2}$, $0.\overline{3}$, $0.\overline{4}$

    $1\times0.\overline{1}$ $2\times0.\overline{1}$ $3\times0.\overline{1}$ $4\times0.\overline{1}$

c. Using the pattern, you should have predicted

$$\underset{\substack{\uparrow \\ 5\times 0.\bar{1}}}{\frac{5}{9}=0.\bar{5}},\quad \underset{\substack{\uparrow \\ 6\times 0.\bar{1}}}{\frac{6}{9}=0.\bar{6}},\quad \underset{\substack{\uparrow \\ 7\times 0.\bar{1}}}{\frac{7}{9}=0.\bar{7}},\quad \underset{\substack{\uparrow \\ 8\times 0.\bar{1}}}{\frac{8}{9}=0.\bar{8}}$$

11. a. $\frac{1}{99}=0.\overline{01},\ \frac{2}{99}=0.\overline{02},\ \frac{3}{99}=0.\overline{03},\ \frac{4}{99}=0.\overline{04},\ \frac{5}{99}=0.\overline{05}$

b.

$$\underset{\substack{\uparrow \\ 1\times 0.\overline{01}}}{0.\overline{01}},\quad \underset{\substack{\uparrow \\ 2\times 0.\overline{01}}}{0.\overline{02}},\quad \underset{\substack{\uparrow \\ 3\times 0.\overline{01}}}{0.\overline{03}},\quad \underset{\substack{\uparrow \\ 4\times 0.\overline{01}}}{0.\overline{04}},\quad \underset{\substack{\uparrow \\ 5\times 0.\overline{01}}}{0.\overline{05}}$$

c. Using the pattern, you should have predicted

$$\underset{\substack{\uparrow \\ 13\times 0.\overline{01}}}{\frac{13}{99}=0.\overline{13}},\quad \underset{\substack{\uparrow \\ 23\times 0.\overline{01}}}{\frac{23}{99}=0.\overline{23}},\quad \underset{\substack{\uparrow \\ 47\times 0.\overline{01}}}{\frac{47}{99}=0.\overline{47}},\quad \underset{\substack{\uparrow \\ 68\times 0.\overline{01}}}{\frac{68}{99}=0.\overline{68}},\quad \underset{\substack{\uparrow \\ 94\times 0.\overline{01}}}{\frac{94}{99}=0.\overline{94}}$$

12. a. $2.\bar{7}=2\frac{7}{9}$

b. $0.\overline{14}=\frac{14}{99}$

c. $3.\overline{26}=3\frac{26}{99}$

d. $1.\bar{8}=1\frac{8}{9}$

13. Method 1: Using Long or Short Division

$$\begin{array}{r} 5.5 \\ 36\overline{)198.0} \\ \underline{180} \\ 18\ 0 \\ \underline{18\ 0} \\ 0 \end{array} \qquad\qquad \begin{array}{r} 5.\ \ 5 \\ 36\overline{)198.{}_{18}0} \end{array}$$

The price of a ticket was \$5.50.

Method 2: Using a Calculator

1 9 8 ÷ 3 6 =

5.5

The price of a ticket was \$5.50.

14. Method 1: Using Long or Short Division

$$\begin{array}{r} 333\ \text{R}4 \\ 12\overline{)4000} \\ \underline{36} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array} = 333\tfrac{4}{12}\ \textbf{or}\ 333\tfrac{1}{3} \qquad\qquad \begin{array}{r} 333\ \text{R}4 \\ 12\overline{)4{}_{4}0{}_{4}00} \end{array}$$

The Board receives $333\frac{1}{3}$ dozen eggs.

Method 2: Using a Calculator

4 0 0 0 ÷ 1 2 =

333.33333

The Board receives $333.\overline{3}$ dozen eggs.

15. a.

$$\begin{array}{r} 55 \text{ R15} \\ 24\overline{)1335} \\ \underline{120} \\ 135 \\ \underline{120} \\ 15 \end{array}$$

$$\begin{array}{r} 5\ \ 5 \text{ R15} \\ 24\overline{)133_{13}5} \end{array}$$

1 3 3 5 ÷ 2 4 =

55.625

The teens needed 56 boxes.

b. There were 55 full boxes.

c. There were 15 bottles in the partly filled box.

Note: If you used a calculator for question 15, you could have found the answer to 15.c. by pressing these keys.

1 3 3 5 ÷ 2 4 =

55.625 ← Find the quotient.

− 5 5 =

0.625 ← Subtract the whole number part of the quotient.

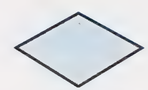
× 2 4 =

15. ← Multiply by the divisor.

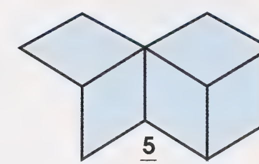
This is the remainder.

Section 1: Activity 4

1. a.

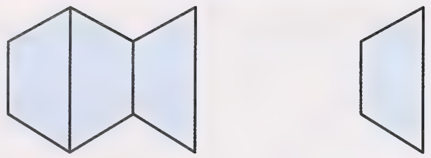


$\frac{1}{6}$



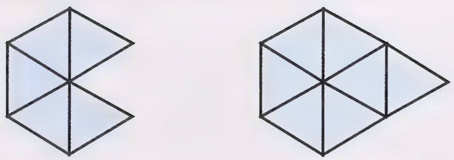
$\frac{5}{6}$

$\frac{1}{6} < \frac{5}{6}$

b. 

$\frac{3}{4}$ $\frac{1}{4}$

$\frac{3}{4} > \frac{1}{4}$

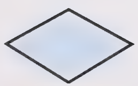
c. 

$\frac{5}{12}$ $\frac{7}{12}$

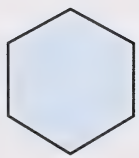
$\frac{5}{12} < \frac{7}{12}$

2. a. $\frac{5}{8} > \frac{3}{8}$ b. $\frac{9}{7} < \frac{11}{7}$ c. $\frac{7}{10} < \frac{9}{10}$

3. a.

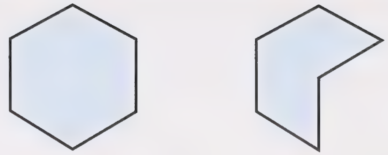


$\frac{1}{6}$



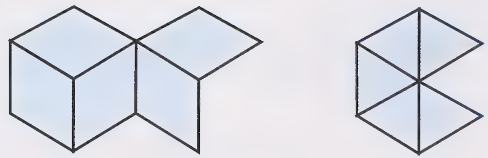
$\frac{1}{2}$

$\frac{1}{6} < \frac{1}{2}$

b. 

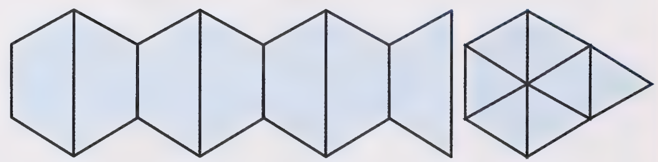
$\frac{1}{2}$ $\frac{1}{3}$

$\frac{1}{2} > \frac{1}{3}$

c. 

$\frac{5}{6}$ $\frac{5}{12}$

$\frac{5}{6} > \frac{5}{12}$

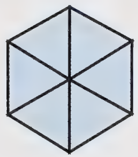
d. 

$\frac{7}{4}$ $\frac{7}{12}$

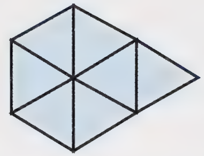
$\frac{7}{4} > \frac{7}{12}$

4. a. $\frac{4}{11} < \frac{4}{5}$ b. $\frac{3}{8} > \frac{3}{10}$ c. $\frac{7}{2} > \frac{7}{3}$

5. a.



$\frac{1}{2} = \frac{6}{12}$

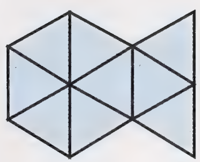


$\frac{7}{12}$

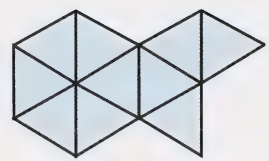
$\frac{6}{12} < \frac{7}{12}$

$\therefore \frac{1}{2} < \frac{7}{12}$

b.



$\frac{3}{4} = \frac{9}{12}$

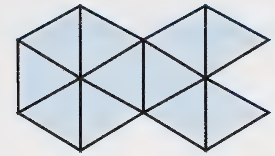


$\frac{5}{6} = \frac{10}{12}$

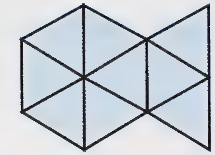
$\frac{9}{12} < \frac{10}{12}$

$\therefore \frac{3}{4} < \frac{10}{12}$

c.



$\frac{11}{12}$



$\frac{3}{4} = \frac{9}{12}$

$\frac{11}{12} > \frac{9}{12}$

$\therefore \frac{11}{12} > \frac{3}{4}$

6. a. $\frac{2}{5} = \frac{14}{35}$; $\frac{3}{7} = \frac{15}{35}$

$\frac{14}{35} < \frac{15}{35}$

$\therefore \frac{2}{5} < \frac{3}{7}$

b. $\frac{7}{9}$; $\frac{2}{3} = \frac{6}{9}$

$\frac{7}{9} > \frac{6}{9}$

$\therefore \frac{7}{9} > \frac{2}{3}$

c. $\frac{3}{4} = \frac{15}{20}$; $\frac{4}{5} = \frac{16}{20}$

$\frac{15}{20} < \frac{16}{20}$

$\therefore \frac{3}{4} < \frac{4}{5}$

d. $\frac{5}{6} = \frac{10}{12}$; $\frac{11}{12}$

$\frac{10}{12} < \frac{11}{12}$

$\therefore \frac{5}{6} < \frac{11}{12}$

7. $\frac{1}{2} = \frac{2}{4};\ \frac{3}{4}$

$\frac{2}{4} < \frac{3}{4}$

$\therefore\ \frac{1}{2} < \frac{3}{4}$

Darwin took more time.

8. $\frac{1}{2} = \frac{12}{24};\ \frac{5}{8} = \frac{15}{24};\ \frac{2}{3} = \frac{16}{24}$

$\frac{12}{24} < \frac{15}{24} < \frac{16}{24}$

$\therefore\ \frac{1}{2} < \frac{5}{8} < \frac{2}{3}$

The biggest diamond of the three is the $\frac{2}{3}$-carat diamond.

9. a. 8 . 1 5

same different

8 . 0 5

1 tenth > 0 tenths

$\therefore$ 8.15 > 8.05

b. 1 . 0 3

same same different

1 . 0 2

3 hundredths > 2 hundredths

$\therefore$ 1.03 > 1.02

c. 0 . 7

same different

0 . 0 7

7 tenths > 0 tenths

$\therefore$ 0.7 > 0.07

d. 5 . 3 2 1

different

4 . 3 2 1

5 ones > 4 ones

$\therefore$ 5.321 > 4.321

e. 0 . 0 6

same different

0 . 1

0 tenths < 1 tenth

$\therefore$ 0.06 < 0.1

f. 0 . 0 2 0

same different

0 . 3

0 tenths < 3 tenths

$\therefore$ 0.020 < 0.3

10. a. $\frac{7}{8} = 0.875$

$0.875 > 0.75$

$\therefore\ \frac{7}{8} > 0.75$

b. $\frac{5}{8} = 0.625$

$0.625 > 0.6$

$\therefore\ \frac{5}{8} > 0.6$

c. $\frac{2}{3} = \frac{20}{30}$; $0.6 = \frac{6}{10} = \frac{18}{30}$

$\frac{20}{30} > \frac{18}{30}$

$\therefore \frac{2}{3} > 0.6$

d. $\frac{1}{7} = \frac{10}{70}$; $0.2 = \frac{2}{10} = \frac{14}{70}$

$\frac{10}{70} < \frac{14}{70}$

$\therefore \frac{1}{7} < 0.2$

11. **a.** $\frac{1}{2} = \frac{12}{24}$; $\frac{2}{3} = \frac{16}{24}$; $\frac{1}{4} = \frac{6}{24}$; $\frac{5}{8} = \frac{15}{24}$

$\frac{16}{24} > \frac{15}{24} > \frac{12}{24} > \frac{6}{24}$

$\therefore \frac{2}{3} > \frac{5}{8} > \frac{1}{2} > \frac{1}{4}$

So, the set of numbers in descending order is

$\left\{\frac{2}{3}, \frac{5}{8}, \frac{1}{2}, \frac{1}{4}\right\}$.

b. $\frac{3}{8} = \frac{375}{1000}$; $0.4 = \frac{4}{10} = \frac{400}{1000}$; $\frac{1}{2} = \frac{5}{10} = \frac{500}{1000}$; $0.75 = \frac{75}{100} = \frac{750}{1000}$

$\frac{750}{1000} > \frac{500}{1000} > \frac{400}{1000} > \frac{375}{1000}$

$\therefore 0.75 > \frac{1}{2} > 0.4 > \frac{3}{8}$

So, the set of numbers in descending order is

$\left\{0.75, \frac{1}{2}, 0.4, \frac{3}{8}\right\}$.

Now Try This

12. You may find it helpful to make an organized list of possibilities. Then test each to see if it works.

Possible Positions	Test	Yes/No
Three firsts	5+5+5=15	no
Two firsts and one second	5+5+3=13	no
One first and three seconds	5+3+3+3=11	no
Four seconds	3+3+3+3=12	yes

Jacinda must have won 4 second-place ribbons.

Section 1: Follow-up Activities

Extra Help

1. **a.** $0.\underset{\uparrow}{01} = \frac{1}{1\,00}$ ← two zeros

two decimal places

b. $0.\underset{\uparrow}{239} = \frac{239}{1\,000}$ ← three zeros

three decimal places

c. $5.\underset{\uparrow}{14} = 5\frac{14}{1\,00}$ ← two zeros

two decimal places

d. $6.\underset{\uparrow}{005} = 6\frac{5}{1\,000}$ ← three zeros

three decimal places

e. $9.\underset{\uparrow}{32} = 9\frac{32}{1\,00}$ ← two zeros

two decimal places

f. $0.\underset{\uparrow}{037} = \frac{37}{1\,000}$ ← three zeros

three decimal places

2. a. $\dfrac{7}{8}=\dfrac{875}{1\ 000}$ ← three zeros

$=0.\ 875$ ← three decimal places

b. $\dfrac{21}{25}=\dfrac{84}{1\ 00}$ ← two zeros

$=0.\ 84$ ← two decimal places

c. $1\dfrac{3}{8}=1\dfrac{375}{1\ 000}$ ← three zeros

$=1.\ 375$ ← three decimal places

d. $3\dfrac{3}{16}=3\dfrac{1875}{1\ 0\ 000}$ ← four zeros

$=3.\ 1875$ ← four decimal places

e. $2\dfrac{7}{125}=2\dfrac{56}{1\ 000}$ ← three zeros

$=2.\ 056$ ← three decimal places

f. $\dfrac{39}{40}=\dfrac{975}{1\ 000}$ ← three zeros

$=0.\ 975$ ← three decimal places

3. a. $\dfrac{4}{10}=\dfrac{\overset{1}{\cancel{2}}\times 2}{\underset{1}{\cancel{2}}\times 5}$

$=\dfrac{2}{5}$

b. $\dfrac{6}{15}=\dfrac{2\times\overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}}\times 5}$

$=\dfrac{2}{5}$

c. $\dfrac{27}{15}=\dfrac{3\times 3\times\overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}}\times 5}$

$=\dfrac{9}{5}$

d. $\dfrac{32}{24}=\dfrac{2\times 2\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times 3}$

$=\dfrac{4}{3}$

e. $\dfrac{16}{36}=\dfrac{2\times 2\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times 3\times 3}$

$=\dfrac{4}{9}$

f. $\dfrac{18}{42}=\dfrac{\overset{1}{\cancel{2}}\times\overset{1}{\cancel{3}}\times 3}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{3}}\times 7}$

$=\dfrac{3}{7}$

Enrichment

1.

Fraction	Fraction with the Denominator Expressed As a Product of Prime Factors	Equivalent Decimal Number	Type of Decimal Number
$\frac{7}{8}$	$\frac{7}{2\times 2\times 2}$	0.875	terminating
$\frac{3}{20}$	$\frac{3}{2\times 2\times 5}$	0.15	terminating
$\frac{4}{5}$	$\frac{4}{5}$	0.8	terminating
$\frac{1}{50}$	$\frac{1}{2\times 5\times 5}$	0.02	terminating
$\frac{9}{11}$	$\frac{9}{11}$	$0.\overline{81}$	repeating
$\frac{7}{12}$	$\frac{7}{2\times 2\times 3}$	$0.58\overline{3}$	repeating
$\frac{5}{18}$	$\frac{5}{2\times 3\times 3}$	$0.2\overline{7}$	repeating
$\frac{5}{13}$	$\frac{5}{13}$	$0.\overline{384\ 615}$	repeating
$\frac{1}{7}$	$\frac{1}{7}$	$0.\overline{142\ 857}$	repeating

2. **a.** When written as a product of prime factors, all the denominators of basic fractions which convert to terminating decimal numbers only have 2s and 5s.

   b. When written as a product of prime factors, all the denominators of basic fractions which convert to repeating decimal numbers have at least one other factor than 2 or 5.

3. Use this test to answer question 3:

   If the denominator of a basic fraction, when written as a product of prime factors, only has 2s or 5s, it is a terminating decimal. If it has any other factors, it will be equivalent to a repeating decimal.

   a. $\frac{11}{12} = \frac{11}{2\times2\times3}$

   It will convert to a repeating decimal number.

   b. $\frac{1}{16} = \frac{1}{2\times2\times2\times2}$

   It will convert to a terminating decimal.

   c. $\frac{10}{49} = \frac{10}{7\times7}$

   It will convert to a repeating decimal number.

   d. $\frac{9}{40} = \frac{9}{2\times2\times2\times5}$

   It will convert to a terminating decimal.

   e. $\frac{8}{125} = \frac{8}{5\times5\times5}$

   It will convert to a repeating decimal number.

   f. $\frac{2}{81} = \frac{2}{3\times3\times3\times3}$

   It will convert to a repeating decimal number.

4. When you write the denominators as a product of prime factors, you will notice the following pattern:

   - The denominator of $\frac{7}{8}$ has **3** twos and the equivalent decimal number has **3** decimal places.
   - The denominator of $\frac{3}{20}$ has **2** twos and the equivalent decimal number has **2** decimal places.
   - The denominator of $\frac{4}{5}$ has **1** five and the equivalent decimal number has **1** decimal place.
   - The denominator of $\frac{1}{50}$ has **2** fives and the equivalent decimal number has **2** decimal places.

5. Use this test to answer question 5:

   The terminating decimal number equivalent to a basic fraction will have the same number of decimal places as the greatest number of twos or fives in the denominator of the fraction when the denominator is written as a product of prime factors.

   a. $\frac{4}{25} = \frac{4}{5\times5}$

   The equivalent decimal number will have 2 decimal places.

   b. $\frac{5}{8} = \frac{5}{2\times2\times2}$

   The equivalent decimal number will have 3 decimal places.

c. $\dfrac{3}{50} = \dfrac{3}{2 \times 5 \times 5}$

The equivalent decimal number will have 2 decimal places.

d. $\dfrac{1}{40} = \dfrac{1}{2 \times 2 \times 2 \times 5}$

The equivalent decimal number will have 3 decimal places.

e. $\dfrac{1}{200} = \dfrac{1}{2 \times 2 \times 2 \times 5 \times 5}$

The equivalent decimal number will have 3 decimal places.

6. a. The maximum number of digits in the repeating block is 10.
   b. The maximum number of digits in the repeating block is 12.
   c. The maximum number of digits in the repeating block is 17.
   d. The maximum number of digits in the repeating block is 14.

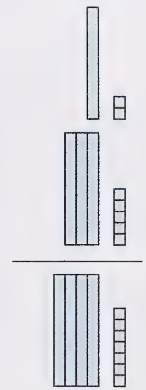
7. Freddy was correct in saying that $\frac{2}{81}$ can be expressed as a repeating decimal number, but he should not have concluded that $\frac{2}{81} = 0.\overline{024\ 691\ 3}$.

   The repeating block could have as many as 80 digits in it; however, Freddy's calculator only shows 7 of these digits. Because Freddy did not know what the other digits are in the places to the right of the decimal point, he could not tell where the pattern begins to repeat.

Section 2: Activity 1

1. a. **Concrete Model**

No regrouping is required.

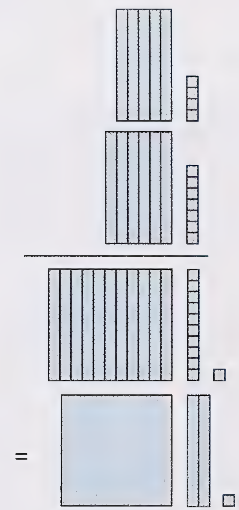


Paper-and-Pencil Method

$$\begin{array}{r} 0.12 \\ +\ 0.35 \\ \hline 0.47 \end{array}$$

b. **Concrete Model**

Regrouping is required.



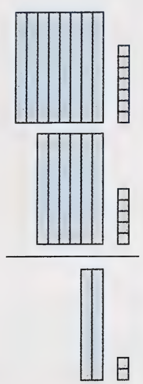
Paper-and-Pencil Method

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 0.54 \\ +\ 0.67 \\ \hline 1.21 \end{array}$$

Regrouping may be done mentally.

2. a. Concrete Model

No regrouping is required.

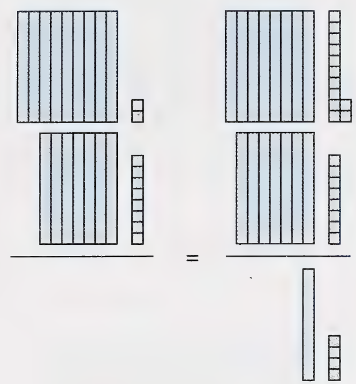


Paper-and-Pencil Method

$$\begin{array}{r} 0.87 \\ -\ 0.65 \\ \hline 0.22 \end{array}$$

b. Concrete Model

Regrouping is required.



Paper-and-Pencil Method

$$\begin{array}{r} {}^{8\,12} \\ 0.\cancel{9}\cancel{2} \\ -\ 0.78 \\ \hline 0.14 \end{array}$$

Regrouping may be done mentally.

3. Estimate

$$\begin{array}{r} 56.2 \\ 57.9 \\ +\ 55.3 \\ \hline \end{array} \doteq \begin{array}{r} 56 \\ 58 \\ +\ 55 \\ \hline 169 \end{array} \textbf{ or } \begin{array}{r} 56 \\ 57 \\ +\ 55 \\ \hline 168 \end{array}$$

Calculated Answer

$$\begin{array}{r} {}^{1\,1} \\ 56.2 \\ 57.9 \\ +\ 55.3 \\ \hline 169.4 \end{array}$$

Regrouping may be done mentally.

The total frontage was 169.4 m.

The answer is reasonable.

4. Estimate

$$\begin{array}{r} 42.50 \\ +\ 42.59 \\ \hline \end{array} \doteq \begin{array}{r} 43 \\ +\ 43 \\ \hline 86 \end{array} \textbf{ or } \begin{array}{r} 40 \\ +\ 40 \\ \hline 80 \end{array}$$

Calculated Answer

$$\begin{array}{r} {}^{1} \\ 42.50 \\ +\ 42.59 \\ \hline 85.09 \end{array}$$

Regrouping may be done mentally.

Wenzil's total time was 85.09 s.

The answer is reasonable.

5. Estimate

$$\begin{array}{r} 133.52 \\ -\ 133.34 \\ \hline \end{array} \doteq \begin{array}{r} 134 \\ -\ 133 \\ \hline 1 \end{array} \textbf{ or } \begin{array}{r} 133 \\ -\ 133 \\ \hline 0 \end{array}$$

Calculated Answer

$$\begin{array}{r} {}^{4\,12} \\ 133.\cancel{5}\cancel{2} \\ -\ 133.34 \\ \hline 0.18 \end{array}$$

Regrouping may be done mentally.

Davis' time was 0.18 s faster.

The answer is reasonable.

6. Estimate

$$\begin{array}{r} 580.23 \\ -\ 530.70 \\ \hline \end{array} \doteq \begin{array}{r} 580 \\ -\ 530 \\ \hline 50 \end{array}$$

Calculated Answer

$$\begin{array}{r} {}^{9} \\ {}^{7\,\cancel{10}\ \ 12} \\ 5\cancel{8}\cancel{0}.\cancel{2}3 \\ -\ 530.70 \\ \hline 49.53 \end{array}$$

Regrouping may be done mentally.

The score was 49.53 points higher.

The answer is reasonable.

7. Estimate

$$\underline{\qquad} \doteq \begin{array}{r} 1 \\ 1 \\ \underline{+\ 1} \\ 3 \end{array} \text{ or } \begin{array}{r} 1 \\ 0 \\ \underline{+\ 1} \\ 2 \end{array}$$

Wendy paid $3.49.

Calculated Answer

$$\begin{array}{r} {}^{1}\\ 1.25 \\ 0.85 \\ \underline{+\ 1.39} \\ 3.49 \end{array}$$

Regrouping may be done mentally.

The answer is reasonable.

8. Estimate

$$\begin{array}{r} 50.00 \\ \underline{-\ 43.29} \end{array} \doteq \begin{array}{r} 50 \\ \underline{-\ 43} \\ 7 \end{array}$$

Roland received $6.71 change.

Calculated Answer

$$\begin{array}{r} {}^{4\,\,9\,\,\,9}_{\,\,\,10\,10\,10} \\ \cancel{5}\cancel{0}.\cancel{0}\cancel{0} \\ \underline{-\ 43.29} \\ 6.71 \end{array}$$

Regrouping may be done mentally.

The answer is reasonable.

9. Estimate

$$\begin{array}{r} 28.20 \\ 32.98 \\ 69.89 \\ 5.99 \\ 29.59 \\ \underline{+\ 11.67} \end{array} \doteq \begin{array}{r} 28 \\ 33 \\ 70 \\ 6 \\ 30 \\ \underline{+\ 12} \\ 179 \end{array} \text{ or } \begin{array}{r} 28 \\ 32 \\ 69 \\ 5 \\ 29 \\ \underline{+\ 11} \\ 174 \end{array}$$

Calculated Answer

2 8 • 2 0 + 3 2 • 9 8 +
6 9 • 8 9 + 5 • 9 9 +
2 9 • 5 9 + 1 1 • 6 7 =

178.32

Hugh's total bill was $178.32.

The answer is reasonable.

10. Estimate

$$\begin{array}{r} 646.38 \\ \underline{-\ 553.34} \end{array} \doteq \begin{array}{r} 646 \\ \underline{-\ 553} \\ 93 \end{array}$$

Calculated Answer

6 4 6 • 3 8 −
5 5 3 • 3 4 =

93.04

The radio mast was 93.04 m taller.

The answer is reasonable.

Now Try This

11. You can use the guess, check, and revise strategy or patterns to solve this problem. There is more than one possible solution.

Original Magic Square

4	9	2
3	5	7
8	1	6

You can multiply each number in the magic square by 5.

New Magic Square

20	45	10
15	25	35
40	5	30

You can add 20 to each number in the magic square.

New Magic Square

24	29	22
23	25	27
28	21	26

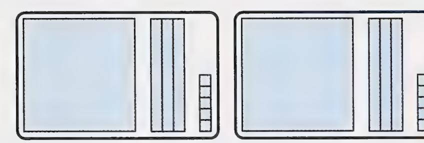
Section 2: Activity 2

1. a. **Step 1:** Model the three groups of 1.12.

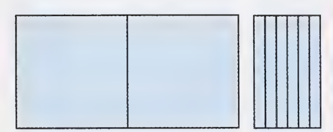
Step 2: Add the groups by combining the hundredths, the tenths, and then the ones.

$\therefore\ 3 \times 1.12 = 3.36$

b. **Step 1:** Model two groups of 1.35.



Step 2: Add the groups by combining the hundredths, the tenths, and then the ones.



Step 3: Regroup. (In this case, 10 small cubes are traded for 1 long.)

$\therefore\ 2 \times 1.35 = 2.7$

2. a. Estimate

$$\begin{array}{r} 0.45 \\ \times\ \ 12 \\ \hline \end{array} \doteq \begin{array}{r} 0.4 \\ \times\ 10 \\ \hline 4 \end{array}$$

Calculated Answer

$$\begin{array}{r} 0.45 \\ \times\ \ 12 \\ \hline 90 \\ 45\ \\ \hline 5.40 \end{array}$$

The stamps cost $5.40.

The answer is reasonable.

b. Estimate

$$\begin{array}{r} 12.98 \\ \times\ \ \ \ 4 \\ \hline \end{array} \doteq \begin{array}{r} 13 \\ \times\ 4 \\ \hline 52 \end{array} \textbf{ or } \begin{array}{r} 12 \\ \times\ 4 \\ \hline 48 \end{array}$$

Calculated Answer

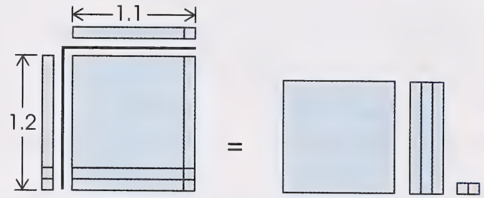
$$\begin{array}{r} 12.98 \\ \times\ \ \ \ 4 \\ \hline 51.92 \end{array}$$

The team paid $51.92.

The answer is reasonable.

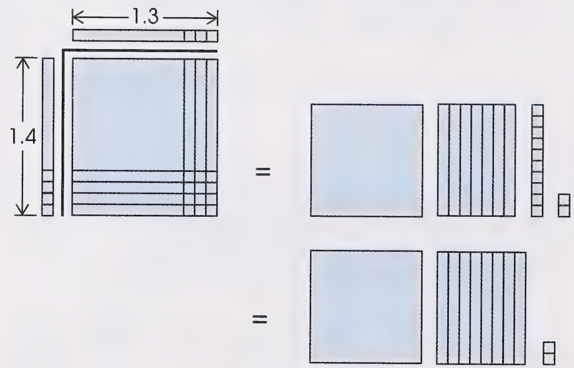
3. **a.** 1.2 and 1.2 **b.** 1.1 and 1.4 **c.** 1.3 and 1.2
d. 1.3 and 2.2 **e.** 1.1 and 2 **f.** 1.1 and 1.1
g. 1.1 and 3.4

4. a.



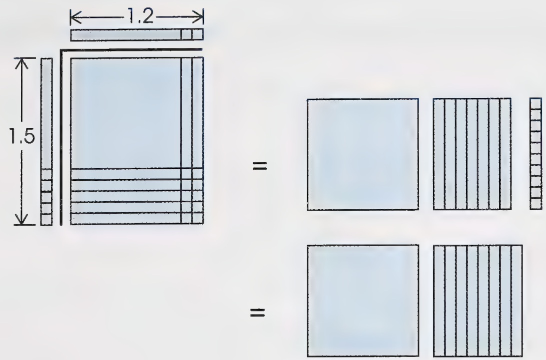
$\therefore\ 1.2 \times 1.1 = 1.32$

b.



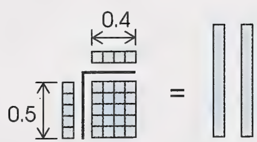
$\therefore\ 1.4 \times 1.3 = 1.82$

c.



$\therefore\ 1.5 \times 1.2 = 1.8$

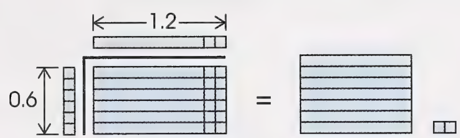
5. a.



$\therefore\ 0.5 \times 0.4 = 0.20$

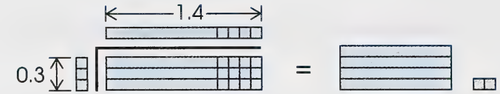
$= 0.2$

b.



$\therefore\ 0.6 \times 1.2 = 0.72$

c.



$\therefore\ 0.3 \times 1.4 = 0.42$

6. Estimate

$$\begin{array}{r} 4.6 \\ \times\ 2.4 \\ \hline \end{array} \doteq \begin{array}{r} 5 \\ \times\ 2 \\ \hline 10 \end{array} \text{ or } \begin{array}{r} 4 \\ \times\ 2 \\ \hline 8 \end{array}$$

Calculated Answer

$$\begin{array}{r} 4.6 \\ \times\ 2.4 \\ \hline 184 \\ 92 \\ \hline 11.04 \end{array}$$

The answer is reasonable.

The mass of a cubic centimetre of gold would be 11.04 g.

7. a. Estimate

$$\begin{array}{r} 0.05 \\ \times\ 7.5 \\ \hline \end{array} \doteq \begin{array}{r} 0.05 \\ \times\ 8 \\ \hline 0.40 \end{array} \text{ or } \begin{array}{r} 0.05 \\ \times\ 7 \\ \hline 0.35 \end{array}$$

Calculated Answer

$$\begin{array}{r} 0.05 \\ \times\ 7.5 \\ \hline 25 \\ 35 \\ \hline 0.375 \end{array} \text{ or } \begin{array}{r} 7.5 \\ \times\ 0.05 \\ \hline 0.375 \end{array}$$

The answer is reasonable.

The wire of a coat hanger is about 0.375 mm thick.

b. $\begin{array}{r} 0.05 \\ \times\ \ 0.04 \\ \hline 0.0020 \end{array}$

The exact answer can be calculated mentally, so no estimation is shown.

A red blood cell is about 0.002 mm thick.

Note: The final zero is not needed as a place holder.

8. Estimate

$\begin{array}{r} 0.125 \\ \times\ \ 5.7 \\ \hline \end{array} \doteq \begin{array}{r} 0.1 \\ \times\ \ 5 \\ \hline 0.5 \end{array}$ **or** $\begin{array}{r} 0.1 \\ \times\ \ 6 \\ \hline 0.6 \end{array}$

Calculated Answer

5 • 7 × 0 • 1 2 5 =

0.7125

The answer is reasonable.

The mass of the elephant's skin is about 0.7125 t.

9. Estimate

$\begin{array}{r} 15.3 \\ \times\ 17.5 \\ \hline \end{array} \doteq \begin{array}{r} 20 \\ \times\ 20 \\ \hline 400 \end{array}$ **or** $\begin{array}{r} 10 \\ \times\ 10 \\ \hline 100 \end{array}$

Calculated Answer

1 5 • 3 × 1 7 • 5 =

267.75

The answer is reasonable.

The frog can jump 267.75 cm.

10. Estimate

$\begin{array}{r} 1194 \\ \times\ \ 2.2 \\ \hline \end{array} \doteq \begin{array}{r} 1000 \\ \times\ \ \ \ 2 \\ \hline 2000 \end{array}$

Calculated Answer

1 1 9 4 × 2 • 2 =

2626.8

The answer is reasonable.

The Concorde's cruising speed is 2626.8 km per hour.

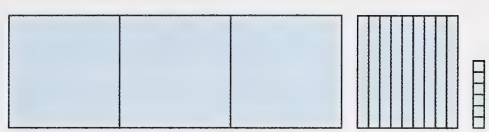
Now Try This

11. Use the guess, check, and revise strategy and logic to solve this problem.

$$
\begin{array}{rrrrr}
 & & 4. & 8 & 9 \\
\times & & & 6. & 9 \\
\hline
 & 4 & 4 & 0 & 1 \\
2 & 9 & 3 & 4 & \\
\hline
3 & 3. & 7 & 4 & 1
\end{array}
$$

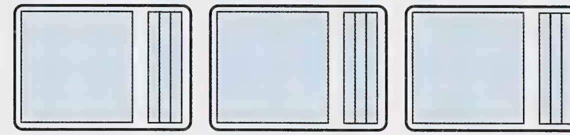
Section 2: Activity 3

1. a. **Step 1:** Model the dividend.

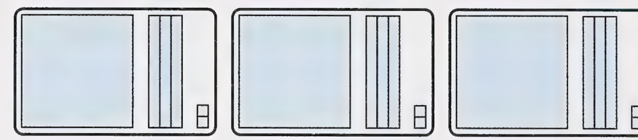


Step 2: Arrange the ones into three groups. There will be 9 tenths and 6 hundredths left over.

Step 3: Arrange the tenths into three groups. There will be 6 hundredths left over.



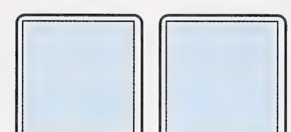
Step 4: Arrange the hundredths into three groups. There will be no remainder.



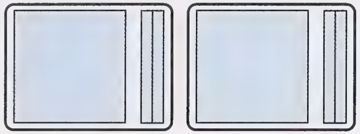
$\therefore\ 3.96 \div 3 = 1.32$

b. **Step 1:** Model the dividend.

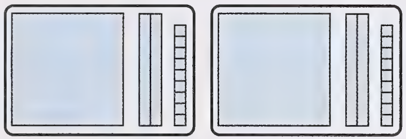
Step 2: Arrange the ones into two groups. There will be 5 tenths and 8 hundredths left over.



Step 3: Trade 1 tenth for 10 hundredths. Then arrange the remaining 4 tenths into two groups. There will be 18 hundredths left over.



Step 4: Arrange the remaining 18 hundredths into two groups.



$\therefore\ 2.58 \div 2 = 1.29$

2. a. **Estimate**

$96.4 \div 8 \doteq 96 \div 8$
$\doteq 12$

Calculated Answer

$$\begin{array}{r} 12.05 \\ 8\overline{)96.40} \\ \underline{8} \\ 16 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

A zero needs to be added to the dividend. This will not change the value.

This car can be driven 12.05 km on 1 L of gasoline.

b. **Estimate**

$36.25 \div 5 \doteq 35 \div 5$
$\doteq 7$

Calculated Answer

$$\begin{array}{r} 7.25 \\ 5\overline{)36.25} \\ \underline{35} \\ 12 \\ \underline{10} \\ 25 \\ \underline{25} \\ 0 \end{array}$$

Janice works 7.25 h each day.

The answer is reasonable.

c. **Estimate**

$123.2 \div 77 \doteq 140 \div 70$
$\doteq 2$

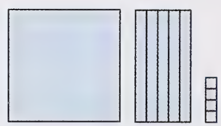
Calculated Answer

$$\begin{array}{r} 1.6 \\ 77\overline{)123.2} \\ \underline{77} \\ 462 \\ \underline{462} \\ 0 \end{array}$$

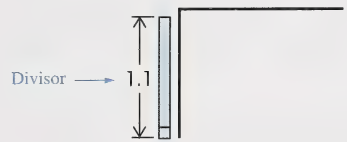
A quarter is 1.6 mm thick.

The answer is reasonable.

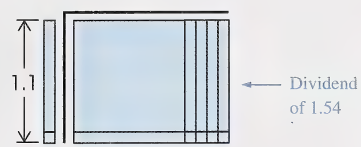
3. a. **Step 1:** Model the dividend of 1.54.



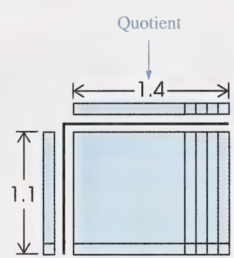
Step 2: Because the divisor is 1.1, there will be 1.1 rows in the rectangular array. Show this.



Step 3: Rearrange the model of the dividend so that a rectangular array with 1.1 rows is formed.



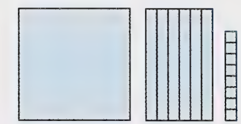
Step 4: To calculate the quotient, determine the number of columns in the rectangular array. You may use 1 long and 4 small cubes to show this.



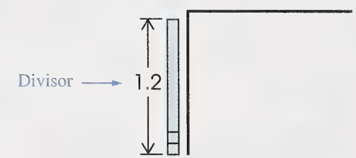
There are 1.4 columns.

$\therefore\ 1.54 \div 1.1 = 1.4$

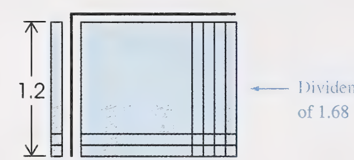
b. Step 1: Model the dividend of 1.68.



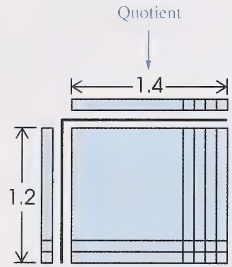
Step 2: Because the divisor is 1.2, there will be 1.2 rows in the rectangular array. Show this.



Step 3: Rearrange the model of the dividend so that a rectangular array with 1.2 rows is formed.



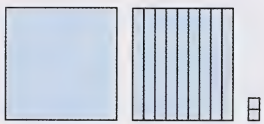
Step 4: To calculate the quotient, determine the number of columns in the rectangular array. You may use 1 long and 4 small cubes to show this.



There are 1.4 columns.

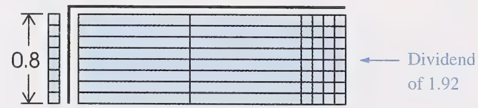
$\therefore\ 1.68 \div 1.2 = 1.4$

4. a. Step 1: Model the quotient of 1.92.

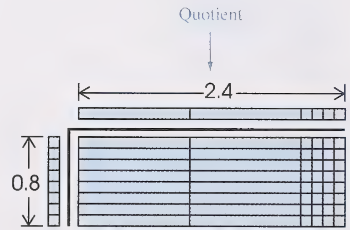


Step 2: Because the divisor is 0.8, there will be 0.8 rows in the rectangular array. Show this.

Step 3: Rearrange the model of the dividend so that a rectangular array with 0.8 rows is formed. To do this you must trade 1 flat for 10 longs and 3 longs for 30 small cubes.



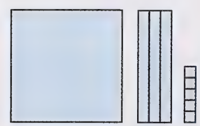
Step 4: To calculate the quotient, determine the number of columns in the rectangular array. You may use 1 long and 6 small cubes to show this.



There are 2.4 columns.

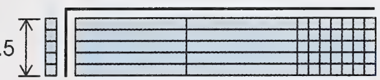
$\therefore\ 1.92 \div 0.8 = 2.4$

b. Step 1: Model the dividend of 1.35.

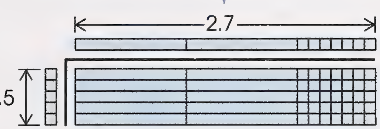


Step 2: Because the divisor is 0.5, there will be 0.5 rows in the rectangular array. Show this.

Step 3: Rearrange the model of the dividend so that a rectangular array with 0.5 rows is formed. To do this you must trade 1 flat for 10 longs and 3 longs for 30 small cubes.



Step 4: To calculate the quotient, determine the number of columns in the rectangular array. You may use 2 longs and 7 small cubes to show this.



There are 2.7 rows.

$\therefore\ 1.35 \div 0.5 = 2.7$

5. a. **Estimate**

$7.5 \div 1.5 \doteq 8 \div 2$

$\doteq 4$

In this time, 5 birdhouses can be built.

Calculated Answer

$$1.5\overline{)7.5} = 5. \quad 75,\ 0$$

The answer is reasonable.

b. **Estimate**

$9 \div 1.50 \doteq 10 \div 2$

$\doteq 5$

You can buy 6 tetras.

Calculated Answer

$$1.50\overline{)9.00} = 6. \quad 900,\ 0$$

The answer is reasonable.

c. **Estimate**

$6 \div 0.75 \doteq 6 \div 1$

$\doteq 6$

There are 8 periods in the day.

Calculated Answer

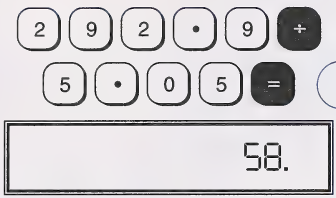
$$0.75\overline{)6.00} = 8. \quad 600,\ 0$$

The answer is reasonable.

6. a. **Estimate**

$292.9 \div 5.05 \doteq 300 \div 5$
$\doteq 60$

Calculated Answer



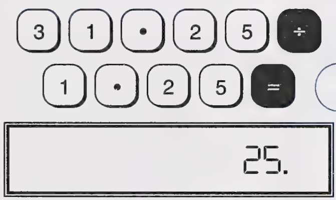
The answer is reasonable.

There are 58 quarters in the pile.

b. **Estimate**

$31.25 \div 1.25 \doteq 31 \div 1$
$\doteq 31$

Calculated Answer



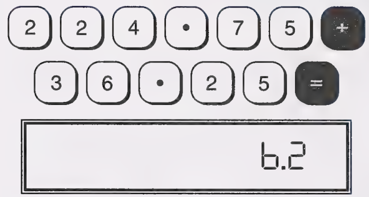
The answer is reasonable.

Marcia's average speed was 25 km/h.

c. **Estimate**

$224.75 \div 36.25 \doteq 200 \div 40$
$\doteq 5$

Calculated Answer



Kim earned $6.20 each hour. The answer is reasonable.

Now Try This

7. Use the guess, check, and revise strategy and logic to solve this problem.

$$\begin{array}{r} 2.32 \\ 25\overline{)58.00} \\ \underline{50} \\ 80 \\ \underline{75} \\ 50 \\ \underline{50} \\ 0 \end{array}$$

Section 2: Follow-up Activities

Extra Help

You may have found it helpful to use graph paper to perform the following calculations. The graph paper helps you line up the digits correctly.

1. Zeros may be used as place holders in these sums and differences.

a.
$$\begin{array}{r} 80.63 \\ +\ 109.80 \\ \hline 190.43 \end{array}$$

b.
$$\begin{array}{r} 5.600 \\ 3.480 \\ 29.600 \\ +\ 0.387 \\ \hline 39.067 \end{array}$$

c.
$$\begin{array}{r} 102.30 \\ -\ 41.17 \\ \hline 61.13 \end{array}$$

d.
$$\begin{array}{r} 8.67 \\ -\ 5.40 \\ \hline 3.27 \end{array}$$

2. a.
$$\begin{array}{rl} 0.68 & \leftarrow \text{2 decimal places} \\ \times\ 7 & \leftarrow \text{0 decimal places} \\ \hline 4.76 & \leftarrow \text{2 decimal places} \end{array}$$

b.
$$\begin{array}{rl} 5.6 & \leftarrow \text{1 decimal place} \\ \times\ 13 & \leftarrow \text{0 decimal places} \\ \hline 168 & \\ 56\ \ & \\ \hline 72.8 & \leftarrow \text{1 decimal place} \end{array}$$

c.
$$\begin{array}{rl} 3.7 & \leftarrow \text{1 decimal place} \\ \times\ 0.6 & \leftarrow \text{1 decimal place} \\ \hline 2.22 & \leftarrow \text{2 decimal places} \end{array}$$

d.
$$\begin{array}{rl} 6.32 & \leftarrow \text{2 decimal places} \\ \times\ 1.8 & \leftarrow \text{1 decimal place} \\ \hline 5056 & \\ 632\ \ & \\ \hline 11.376 & \leftarrow \text{3 decimal places} \end{array}$$

3. a.
$$\begin{array}{r} 1.32 \\ 5\overline{)6.60} \\ \underline{5}\ \ \ \ \\ 16\ \ \\ \underline{15}\ \ \\ 10 \\ \underline{10} \\ 0 \end{array}$$

b.
$$\begin{array}{r} 0.7 \\ 43\overline{)30.1} \\ \underline{301} \\ 0 \end{array}$$

c. $0.09\overline{)47.7} \longrightarrow 9\overline{)4770}$

$$\begin{array}{r} 530 \\ 9\overline{)4770} \\ \underline{45}\ \ \ \\ 27\ \ \\ \underline{27}\ \ \\ 00 \\ \underline{0} \\ 0 \end{array}$$

d. $4.1\overline{)36.9} \longrightarrow 41\overline{)369}$

$$\begin{array}{r} 9 \\ 41\overline{)369} \\ \underline{369} \\ 0 \end{array}$$

Enrichment

1. a.

*	6.98	
*	2.97	
	9.95	TL
	20.00	CT
	10.05	CG

b.

```
* ............................ 5.99
* ............................ 6.99
* ............................ 8.99
                              21.97 TL

                              30.00 CT
                               8.03 CG
```

2. a.

```
* ............................ 4.32
* ............................ 6.41
* ............................ 2.15
                              12.88 TL
```

b.

```
* ............................ 4.15
* ............................ 3.23
* ............................ 1.51
                               8.89 TL
```

c.

```
* ............................ 12.22
* ............................ 5.34
* ............................ 4.41
                               21.97 TL
```

3. a.

```
* ............................ 1.29
* ............................ 2.34
* ............................ 1.22
                               4.85 TL
```

b.

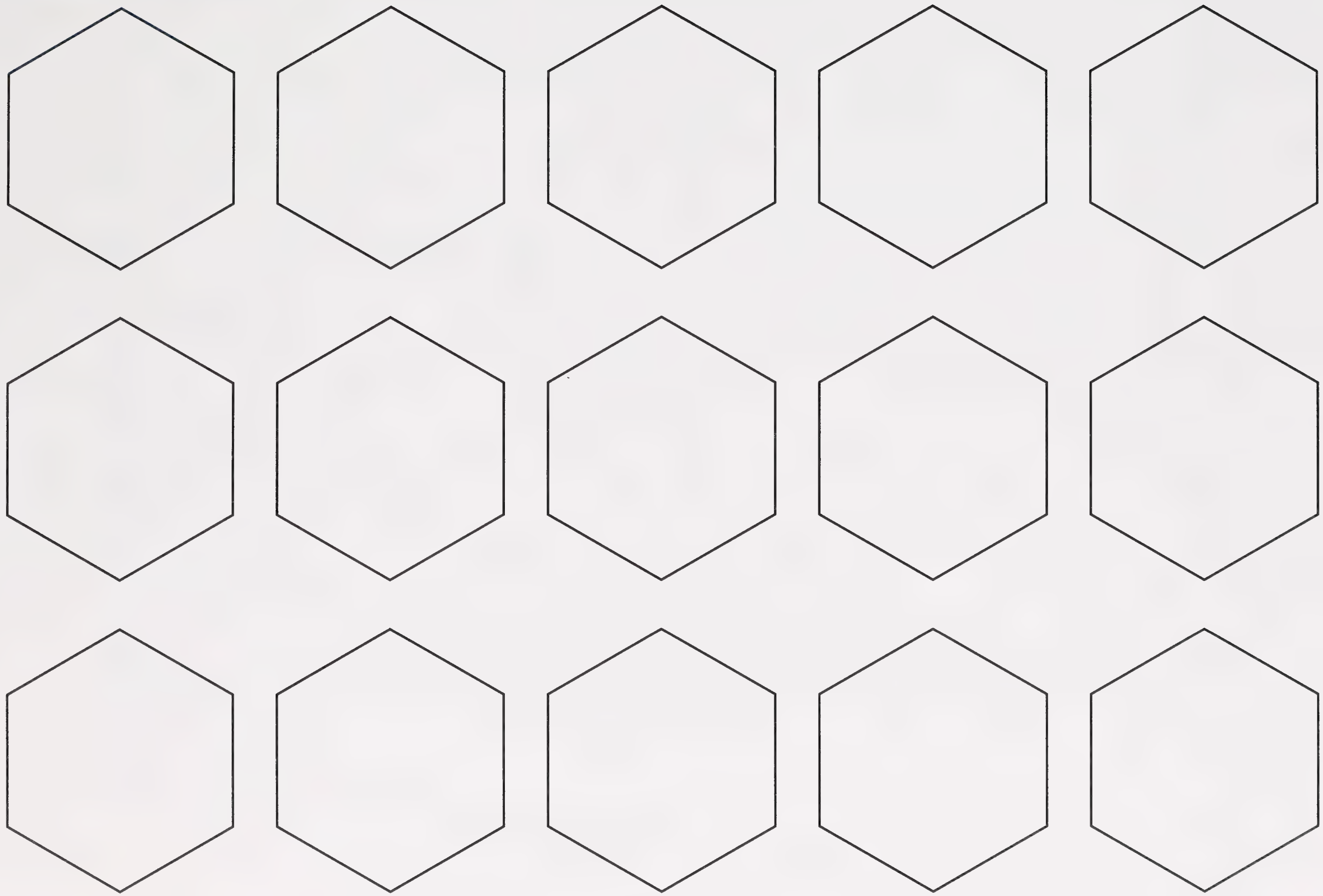
```
* ............................ 1.64
* ............................ 2.28
* ............................ 3.72
                               7.64 TL
```

c.

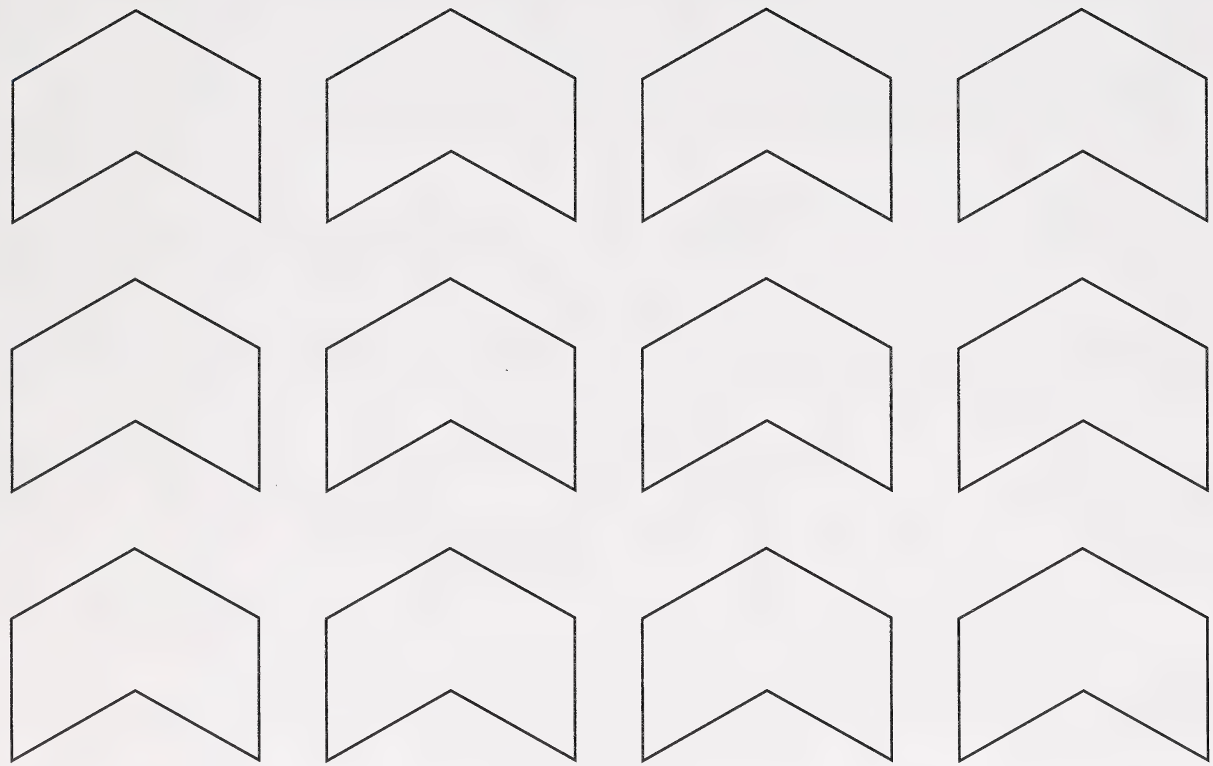
```
* ............................ 4.98
* ............................ 3.99
* ............................ 7.99
                              16.96 TL
```

Pattern Blocks

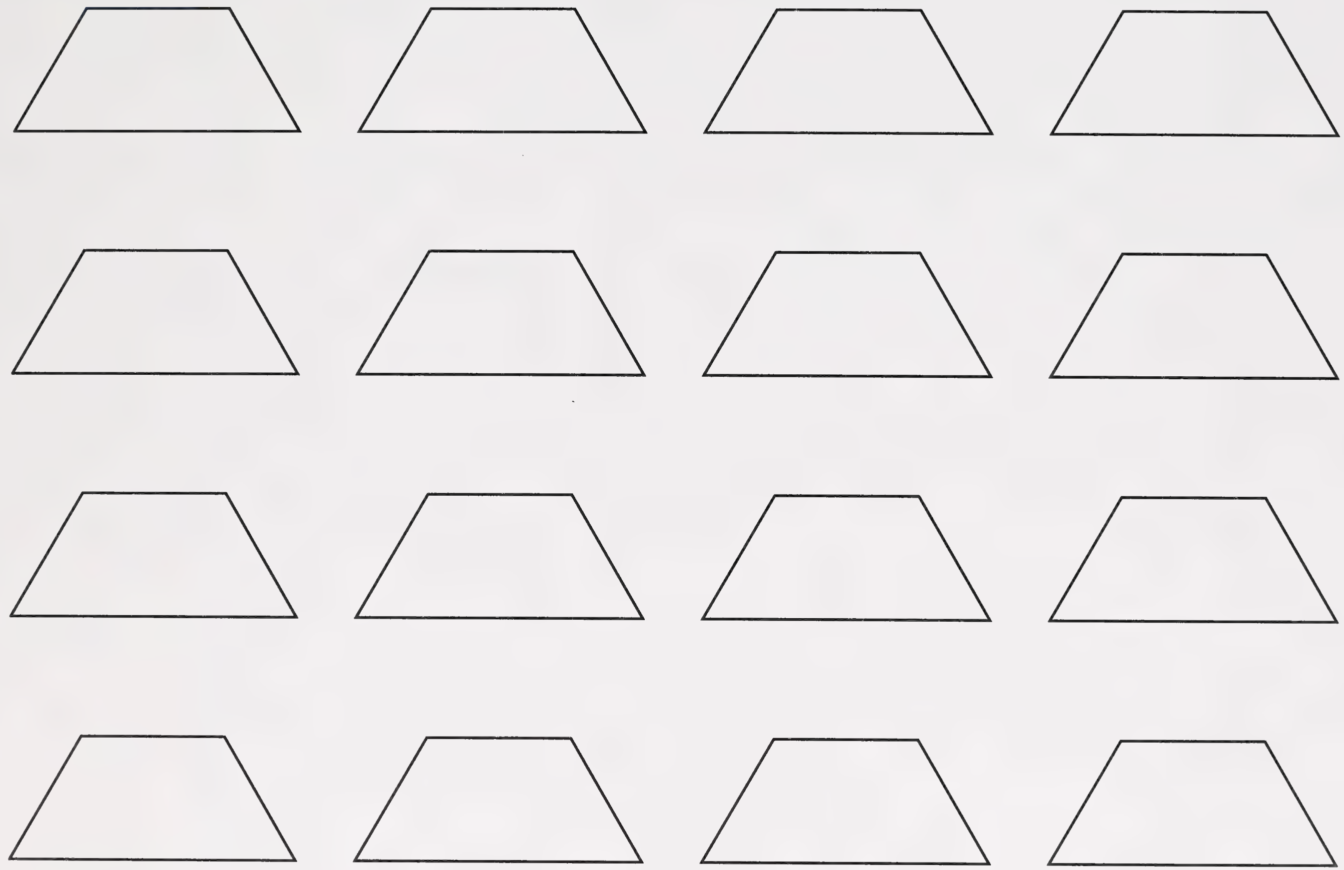
Pattern Blocks



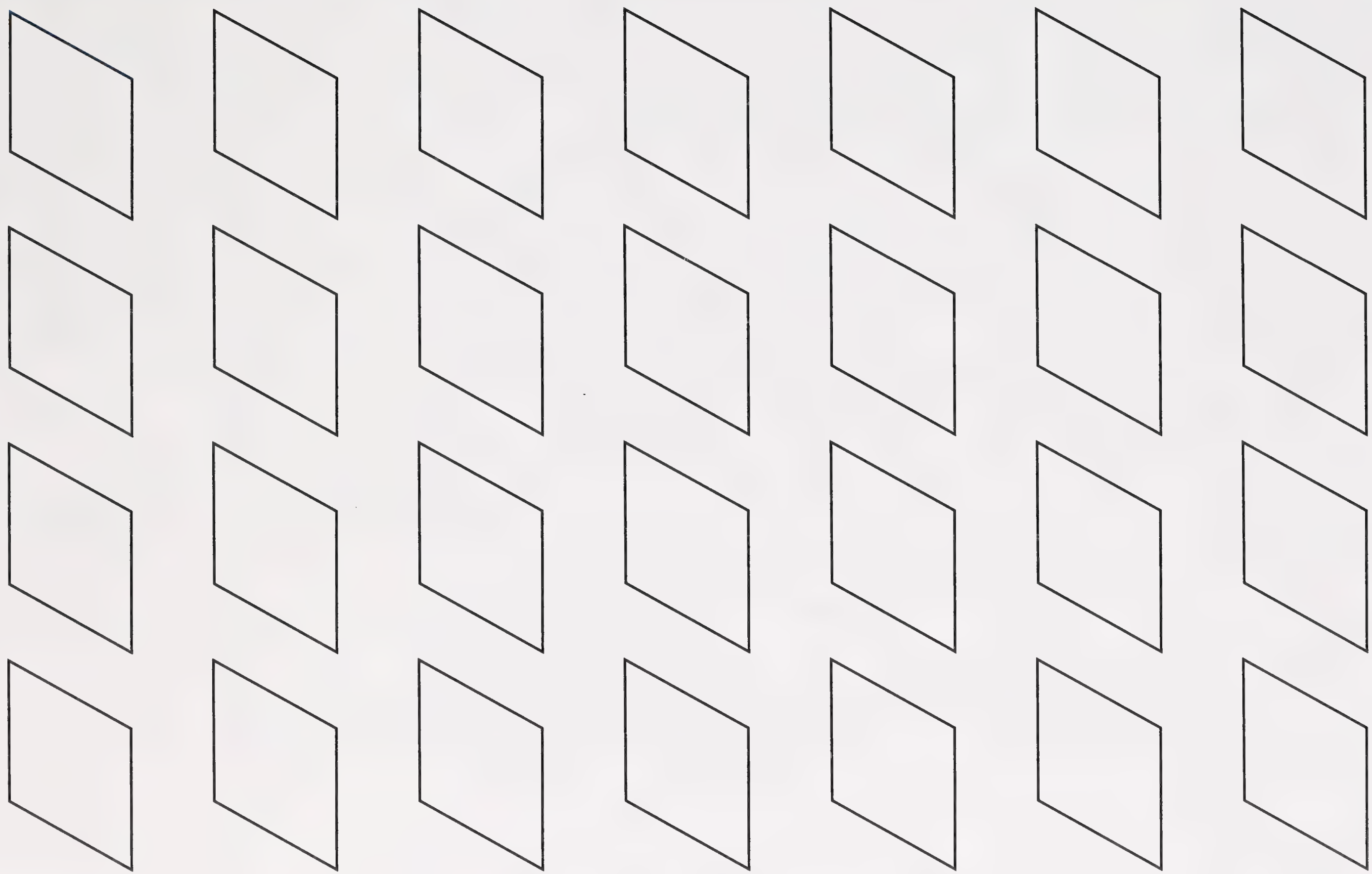
Pattern Blocks



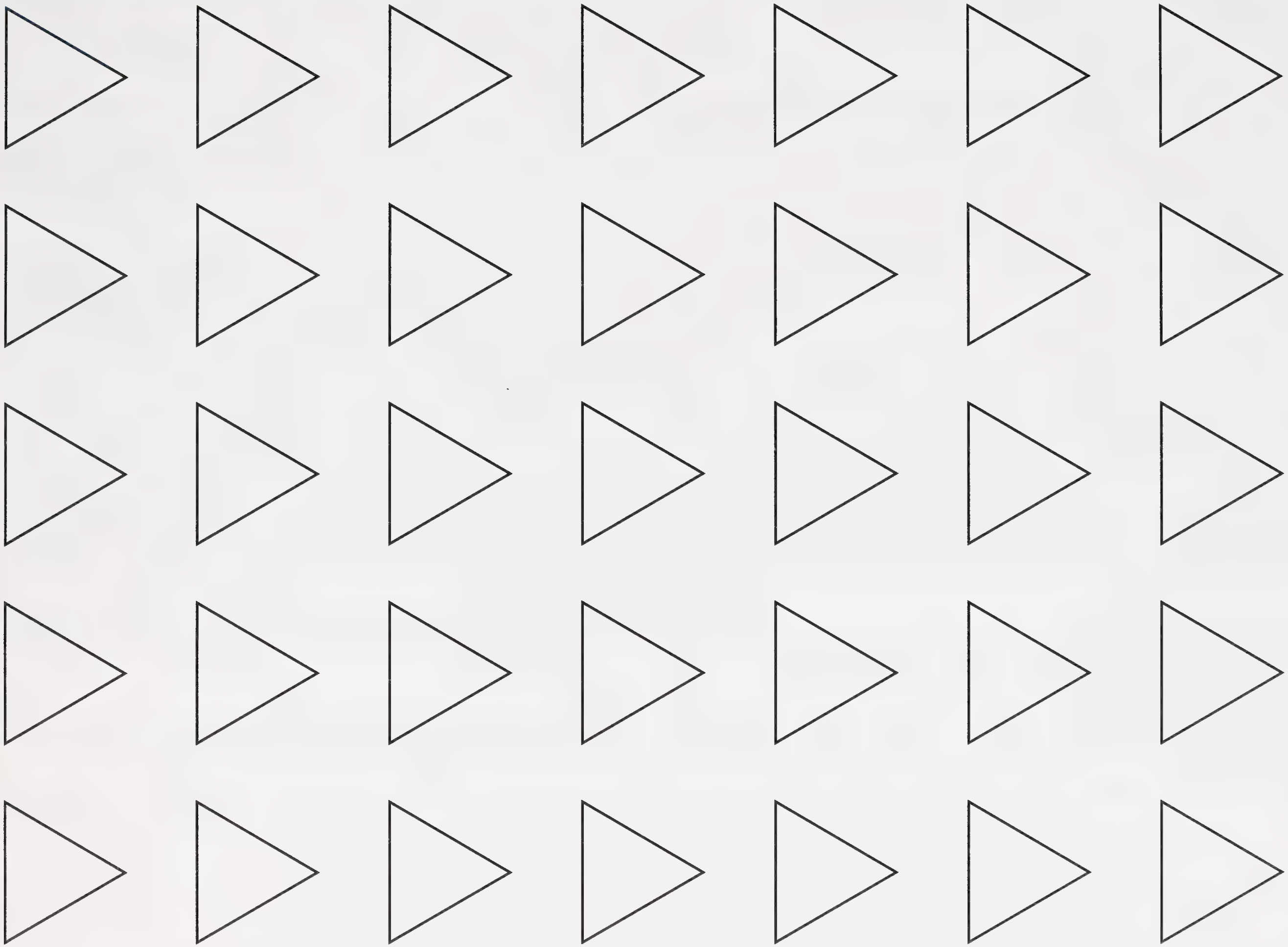
Pattern Blocks



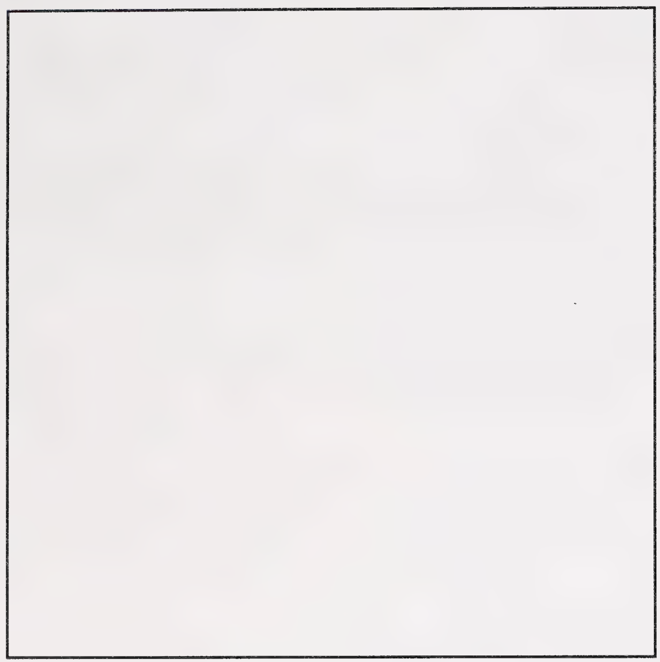
Pattern Blocks

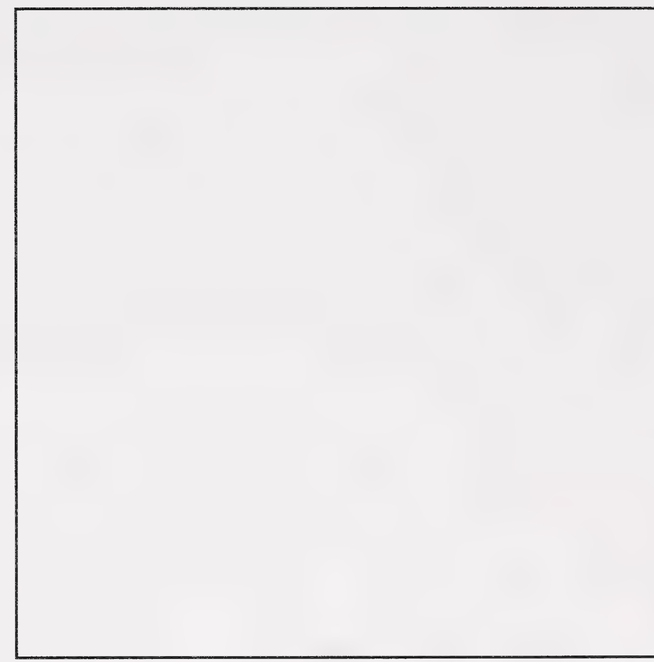


Pattern Blocks

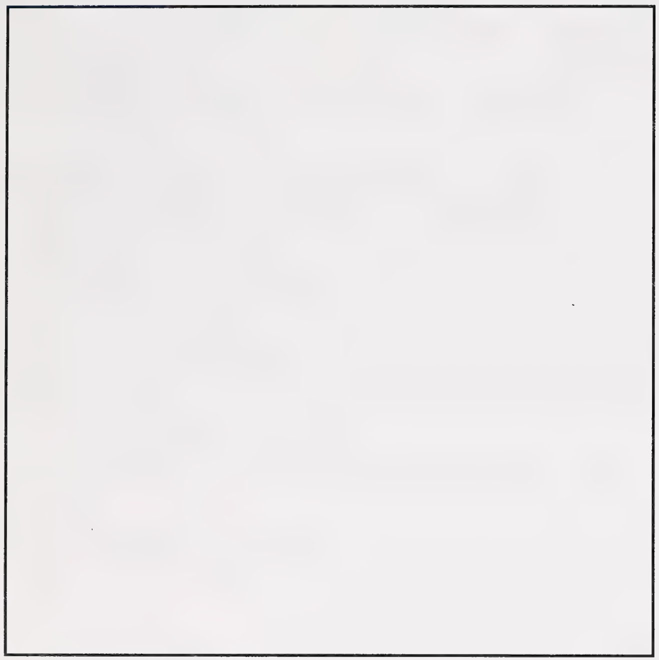


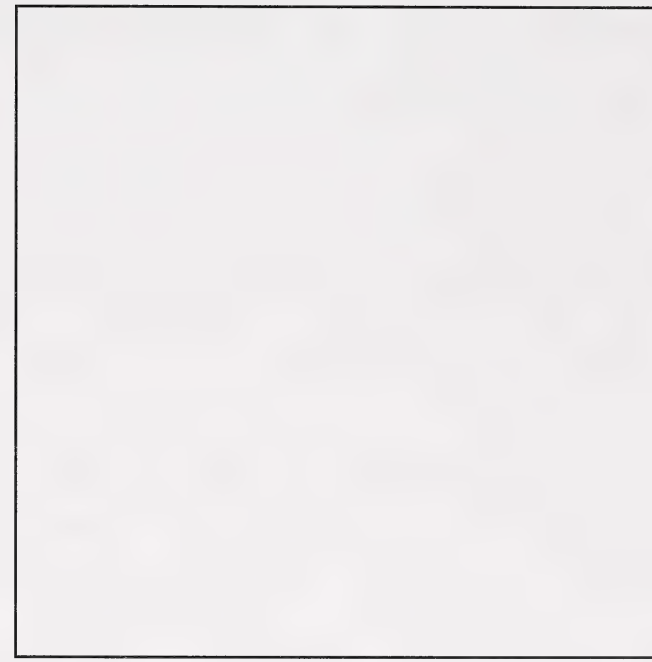
Base Ten Blocks



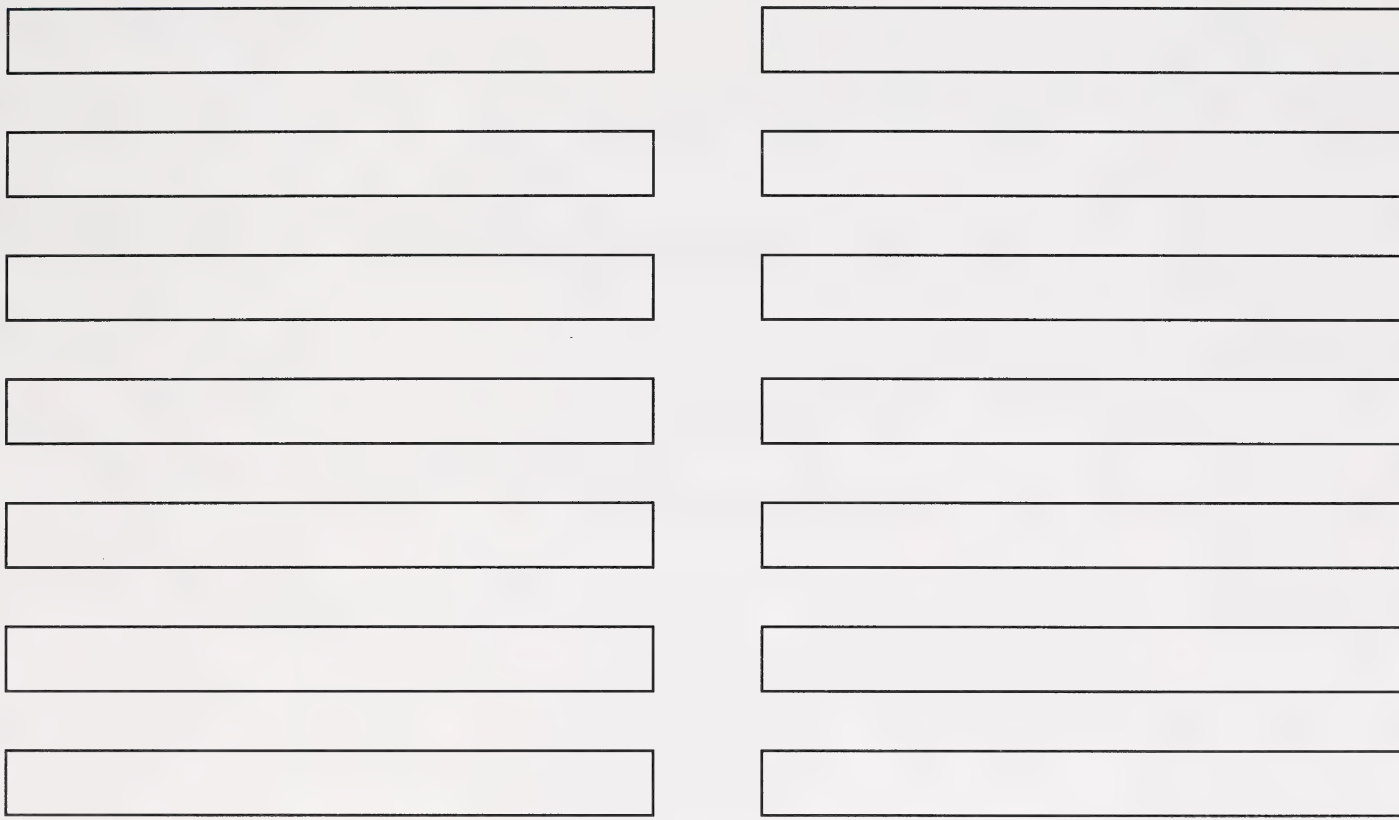


Base Ten Blocks





Base Ten Blocks



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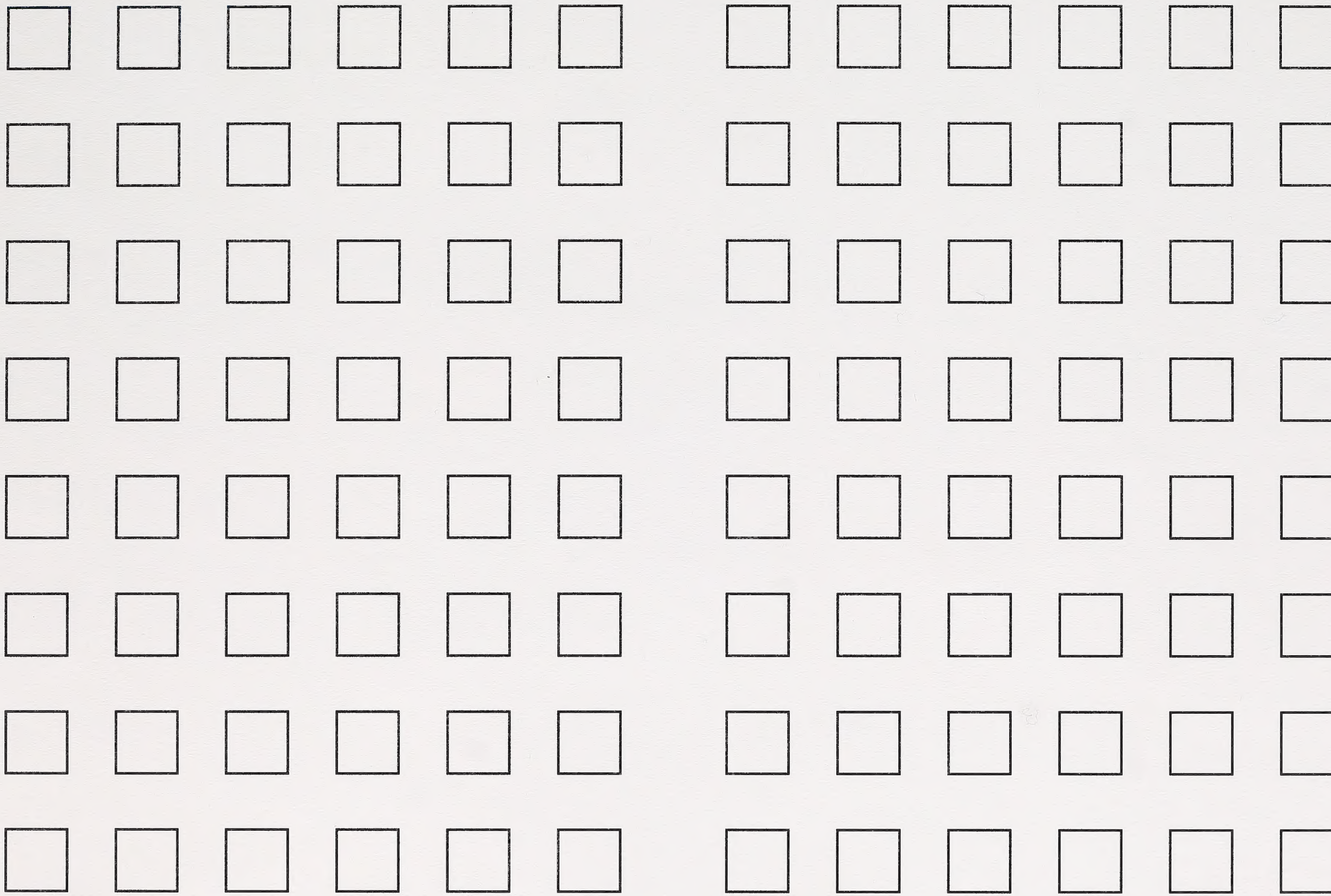

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